Creating effective mathematics lessons can be a challenge for any teacher. Typically, the biggest hurdle for many teachers is motivating secondary students to want to learn and keeping them interested. The key to motivation is designing lessons that are well organized and centered around student engagement. Whether your district requires well-organized lesson plans or not, it is beneficial to have them. Findings from recent brain research indicate that students are mentally primed to learn during the first fifteen minutes of a typical forty-five-minute class period (Sousa 2001). During the middle fifteen minutes of class time, they experience a mental slump and are more apt to daydream. This research suggests a need to restructure the typical mathematics class from reviewing homework during the first ten minutes of class to getting students’ immediate attention by involving them in a problem or activity. One way to design an effective lesson is to use a before-during-after (BDA) format.

WHAT IS A BDA?
BDA is a lesson format that divides the class session into three parts, organizing activities so that students can better understand the meaning behind the lesson.
Draw a picture of a rectangular prism and identify three items in a grocery store that are in the shape of a rectangular prism.

Then, suppose a rectangular prism has a volume of 126 cubic cm. What are the possible dimensions of the prism?

**Fig. 1** DO NOW

![DO NOW](image)

**The During Phase**

The core mathematical content of the lesson is embedded in the *during* phase. This phase engages the students in experiments, explorations, or guided discovery of a concept either individually or in small groups.

For the *during* phase that follows the DO NOW in *figure 1*, groups of four students are each given 20 cm × 20 cm pieces of 1 cm grid paper. Students then cut out equal-size squares from each corner of the larger grid paper, but the side lengths of the smaller square cutouts vary from student to student: 1 cm, 2 cm, 3 cm, . . . , 9 cm. Students fold up the four sides and tape them to form an “open rectangular prism.” Then they find the length, width, height, surface area, and volume of their open prism (*table 1*). The teacher enters the data for all the prisms on a master table while students fill out their own copy of the same table.

Now it is time to make sense of the data. On the board, the teacher sketches the *x*- and *y*-axes of a graph labeled “height of prism” and “volume,” respectively. Students select a representative from their group to tape a prism onto the graph at its correct position (see *fig. 2*). For example, the open rectangular prism with height 9 cm and volume 36 cubic cm would be taped at the point (9, 36). A discussion follows about the maximum volume displayed by the graph, and students are asked whether this would be the maximum volume if nonintegral units were allowed. Students pair-share their results to discuss the maximum volumes they found. The *during* phase engages students in building understanding of the concept.

**Table 1**

**Prism Worksheet**

<table>
<thead>
<tr>
<th>Size of Square Cutout</th>
<th>Length of Rectangular Prism</th>
<th>Width of Rectangular Prism</th>
<th>Height of Rectangular Prism</th>
<th>Surface Area of Open Rectangular Prism</th>
<th>Volume of Rectangular Prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm × 1 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 cm × 2 cm</td>
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<td>.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>9 cm × 9 cm</td>
<td></td>
<td></td>
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</tbody>
</table>

**Fig. 2** Taping prisms to the whiteboard
and in exploring the mathematical connections in the lesson.

The After Phase
The after phase focuses on a reflection on the exploration and provides an opportunity for students to make sense of the mathematical meaning of the lesson. It also promotes further reasoning and problem solving through extensions. For example, the students describe how they found the maximum volume of the rectangular prisms in their notebooks. They are also asked to find the surface area of each rectangular prism and write an explanation of why the 1 cm square cutout prism has the maximum surface area. A discussion follows on how an algebraic representation of the volume would be written: \((20 - 2x)(20 - 2x)x = \text{Volume}\), where \(x\) is the length of the side of the square that was cut out. Students can enter the data points into their calculators and graph the data. Then they can graph the equation that represents the algebraic representation of the volume. Using the cursor, they can find the maximum volume displayed by the data points and compare their solutions to the nonintegral height of the prism that would hold the maximum volume as represented by the algebraic equation.

Extensions such as the following can be posed: What if the original dimensions were 10 cm \(\times\) 10 cm, 30 cm \(\times\) 30 cm, or 10 cm \(\times\) 20 cm? The after phase also provides an opportunity for the students to communicate and summarize their conclusions either in written format or orally, a practice often neglected in mathematics classes.

In addition, the after phase can provide opportunities for the teacher to embed assessments to measure students’ conceptual and mathematical understanding. If student explanations seemed confusing, further questions can be posed to help guide them through their understanding of the content, or several additional examples can be shared and discussed.

HOW TO DESIGN A LESSON USING THE BDA MODEL
Preparing a lesson begins with identification of the big ideas you would like students to understand. Aligning these goals with the state and national standards has become important in light of No Child Left Behind. Once the lesson goals are determined, break the lesson into the three parts.

The lesson should not be three disjointed activities; each phase should support the others. The questions in table 2 can guide you in developing the BDA plan.

Figure 3 shows a precalculus lesson designed using a BDA format. The teacher found that his students could visually identify the increasing, decreasing, and constant portions of graphs and functions but had difficulty expressing them in interval format. The lesson focused on helping students communicate interval notation both orally and in writing.

BENEFIT OF USING A BDA MODEL
The BDA model for lesson design benefits the teacher as well as the students because a well-planned and connected lesson provides a consistent flow of concepts throughout the class period. The students are engaged immediately, provided with

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Teachers’ Guide to Developing a Before-During-After Lesson</th>
</tr>
</thead>
</table>
| **The Before Phase** | • Does it relate to today’s lesson?  
• Is it a 5- to 10-minute activity?  
• Does it grab students’ attention?  
• Does it allow for connections and/or assess prior knowledge?  
• Do students have the opportunity to share their thinking?  
• Does it actively engage students? |
| **The During Phase** | • Is it aligned with academic standards?  
• Does it meet the course/content objectives?  
• Does it reflect a problem-solving approach?  
• Does it promote opportunities for students to communicate their learning?  
• Are students actively talking, reading, writing, and making sense of the mathematics? |
| **The After Phase** | • Does it require application of the new knowledge?  
• Does it assess what the students have learned?  
• Does it provide opportunity for the students to reflect on their learning and make sense of the mathematics? |
### Title
Graphical and Interval Notation

### Essential Questions
- How do you express increasing, decreasing, and constant functions represented in a graph in written interval notation?
- How do you identify the maximum and minimum values in a graph?

### Before Activities: DO NOW!

**Time:** 10 min.

- Students are given the following inequalities and asked to solve each and represent the graphical solutions in interval notation:

  \[
  4(3 - x) < -9 \quad \quad 2x + 3(x + 4) > 11
  \]

- Students will pair-share their results and compare their graphical solutions.
- The teacher will pose questions to the whole class such as: “What approaches did you use to solve the inequalities?” “How does changing the inequality sign affect the solution?” “What does the graphical solution represent?”

### During Activities

**Time:** 25 min.

- Students work in pairs to answer the questions on the Football Stadium handout (see fig. 4). The goal is to have students state the range, or intervals of time, where the number of fans was increasing, decreasing, or constant.
- In groups of four, students share their answers to the questions on the handout. One group will be selected to share its results with the class.
- The class discusses the increasing, decreasing, and constant intervals over time. The discussion should also focus on the maximum and minimum values of the graphs.
- Students will write three to five sentences in their notebook about the relationships they see between the slope of the graph at various points and where the function is increasing, decreasing, and constant. Example sentences: “The game was approximately three hours long.” “The slope of the graph is positive to indicate the number of fans entering the stadium and negative to indicate the number of fans leaving the stadium.” “During the game, there are approximately 44,000 fans in attendance.”
- Students will read their sentences to their partners to compare and later discuss their findings as a class.
- Next, students work in groups of four on the Soup Can problem (see fig. 5) and state the intervals of increasing, decreasing, and constant temperature for the soup. Students will present their graphs and the intervals to the class and explain their solutions.

### After Activities

**Time:** 10 min.

- Students write a four- to five-line summary of how to state the increasing, decreasing, and constant intervals of a graph in their notebooks or as a ticket out the door. For example: “The graph has a positive slope when the soup is heating up in the microwave for three minutes. After that, the temperature cools, so the graph starts declining in a negative slope over the 20-minute period while the soup sat on the counter. The graph showed a constant function at the beginning when the soup can was taken out of the cupboard and the soup was mixed with water at room temperature.”
- Students pair-share their written summaries, and several students are selected to read theirs to the class.

### Assignment
Students write a function story, draw the graph of the story, and state the increasing, decreasing and constant intervals for the function.
opportunities to make sense of the mathematics, and given time to reflect on their learning and summarize or apply their knowledge.

The BDA model helps teachers focus on a strategy-based instructional technique that promotes effective implementation of the lesson. Students are more motivated throughout the class because of the varied activities conducted within one lesson. As research suggests, student learning increases when students are actively engaged and making sense of the mathematics (Hiebert 1999). Students learn new concepts and skills best when they actively construct their own meaning and when they are immediately immersed in a lesson (Sousa 2001, p. 88).

CONCLUSION
The BDA model is already effectively used to engage students in many reading classes. Likewise, mathematics classes designed using this model can promote effective teaching and learning. For mathematics teachers, incorporating a BDA model into a lesson plan can help organize and guide the design of activities to promote and enhance students’ mathematical experiences.

The authors would like to thank Louis Quackenbush of the mathematics department at John Harris High School in Harrisburg, Pennsylvania, for his help in organizing this article.

BIBLIOGRAPHY

Eric took a can of soup from a cupboard and poured the soup into a bowl. He then poured water, also at room temperature, into the bowl and stirred the soup and water together. Next, Eric heated the mixture in a microwave oven. After three minutes, he saw the soup boiling and removed the bowl from the microwave and poured some of the contents into a smaller bowl. Eric ate the soup in the smaller bowl, but the rest of the soup cooled down to room temperature in twenty minutes. Finally, Eric placed the original bowl and its contents in the refrigerator, where the soup continued to cool to refrigerator temperature.

Your mission:

- Construct a graph representing the temperature of the soup over time.
- State the intervals when the temperature of the soup is increasing, decreasing, or staying the same.
- Present and explain your graph and intervals to the class.

Fig. 5 Soup Can problem

JANE MURPHY WILBURNE, jmw41@psu.edu, is an assistant professor of mathematics education at Penn State Harrisburg in Middletown, PA 17057. She is interested in problem solving and student-centered teaching. WINNIE PETERSON, wpeterso@kutztown.edu, teaches mathematics content and methods to preservice elementary teachers at Kutztown University, PA 19530. Both authors are currently involved in the PA High School Coaching Initiative promoting literacy in mathematics instruction.