CALCULUS AND ANALYTICAL GEOMATRY
MTH-101
Assignment No.1

Solution

Question: 1

Find the solution for: $\left| x^2 - 36 \right| = x - 6$.

Solution:

Depending on whether $x^2 - 36$ is $+ve$ or $-ve$, the above eq. Can be witten as:

$x^2 - 36 = +(x - 6)$ Or $x^2 - 36 = -(x - 6)$
$x^2 - 36 = x - 6$ ---------(1) Or $x^2 - 36 = -x + 6$------------(2)
Solving (1) and (2), we have

$x^2 - 36 + x^2 - 36 = x - 6 + (-x + 6)$

$2x^2 - 72 = x - 6 - x + 6$

$2(x^2 - 36) = 0$

$x^2 - 36 = 0$

$x^2 = 36$

$x = \pm 6$

$x = 6, -6$

Question: 2

Find the coordinates of the center and radius of the circle described by the following equation.

$$x^2 + y^2 + 10x - 6y + 18 = 0 .$$

Solution:

$x^2 + y^2 + 10x - 6y + 18 = 0$

$(x^2 + 10x) + (y^2 - 6y) + 18 = 0$

$(x^2 + 10x) + (y^2 - 6y) = -18$

$\therefore$ By completeing square of
$(x^2 + 10x)$ and $(y^2 - 6y)$
So

$x^2+10x=0$ $\quad$ $y^2-6y=0$

$x^2+2(\mathbf{5})x+(5)^2=(5)^2$ $\quad$ $y^2-2(\mathbf{3})x+(3)^2=(3)^2$

$x^2+10x+25=25$ $\quad$ $y^2-6x+9=9$

$(x^2+10x+25)+(y^2-6y+9)=-18+25+9$

$(x+5)^2+(x-3)^2=16$

$(x-(-5))^2+(x-3)^2=(4)^2$

Hence the coordinates of center is $(-5,3)$ and radius of circle is 4.

Question: 3

Consider the functions $f(x)=x^2+5$ and $g(x)=\sqrt{x}$. Find the composite function $(f\circ g)(x)$ and also find the domain of this composite function.

Solution:

$(fog)(x)=f(g(x))=(\sqrt{x})^2+5$

$=x+5$

Domain of g is $[,+\infty)$
Domain of f is $(-\infty,+\infty)$
Domain of $(fog)(x)$ consists of all x $[0,+\infty)$ in such that g (x) lies in $(-\infty,+\infty)$
Hence its domain is $[0,+\infty)$

Question: 4

Evaluate $\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$.

Solution:

Solution #1	Solution #2
$=\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$ By using L.hospital's Rule, we have $=\lim_{x\to 1}\frac{2x+1-0}{2x-1}$ $=\lim_{x\to 1}\frac{2x+1}{2x-1}$ Applying limit, we have	$=\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$ Now x^2+x-2 can be written as $(x+2)(x-1)$, so we have $=\lim_{x\to 1}\frac{(x+2)(x-1)}{x^2-x}$ $=\lim_{x\to 1}\frac{(x+2)(x-1)}{x(x-1)}$ $=\lim_{x\to 1}\frac{(x+2)}{x}$ $=\lim_{x\to 1}\frac{x(1+\frac{2}{x})}{x}$

$= \frac{2(1)+1}{2(1)-1}$ $= \frac{2+1}{2-1}$ $= \frac{3}{1}$ $= 3$	$= \lim_{x \to 1} \frac{(1 + \frac{2}{x})}{1}$ $= \lim_{x \to 1} (1 + \frac{2}{x})$ Applying limit, we have $= 1 + \frac{2}{1}$ $= 1 + 2 = 3$

Question: 5

Check the continuity of following function at $x = \frac{3}{2}$

$$f(x) = |2x - 3|$$

Solution:

$f(x) = |2x - 3|$

Applying Limit, we have

$$\lim_{x \to \frac{3}{2}} f(x) = \lim_{x \to \frac{3}{2}} |2x - 3|$$

$$= \left| \lim_{x \to \frac{3}{2}} 2x - 3 \right|$$

Putting Limit, we have

$$= 2(\frac{3}{2}) - 3$$

$$= 3 - 3$$

$$= 0$$

$$Hence\ Function\ is\ comtinuous\ at\ x = \frac{3}{2}$$