

## **MATHEMATICAL QUALITY OF INSTRUCTION (MQI)**

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This material is based upon work supported by the National Science Foundation under grant numbers EHR-0233456, DUR-0335411 and DRL-0918383 and work supported by the U.S. Department of Education under cooperative agreement number R305C090023.

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## SEGMENT CODES

<b>Classroom Work is Connected to Mathematics</b>	
Score here for whether the focus is on <i>mathematics content</i> during half or more of the segment (3.75 minutes or more total for a 7.5-minute segment).	
No	Yes
<p>Focus for majority of the segment (at least 3.75 minutes for a 7.5-minute segment) is on non-mathematical topics, or student activities that have no clear connections to developing mathematical content.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Gathering or distributing materials, other administrative issues</li> <li>• Disciplinary issues that severely impinge upon instructional time</li> <li>• Students doing an activity (cutting, pasting, coloring) that is not clearly connected to mathematics (“bad reform”)</li> </ul>	<p>Focus is on mathematical content for majority of the segment (at least 3.75 minutes for a 7.5-minute segment).</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Teacher reviews content from a prior lesson</li> <li>• Teacher introduces content</li> <li>• Students practice content</li> <li>• Students work on a warm-up problem while teacher takes attendance</li> </ul>

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## Richness of the Mathematics

This dimension attempts to capture the depth of the mathematics offered to students. Rich mathematics is either a) focused on the meaning of facts and procedures OR b) focused on key mathematical practices. Rich mathematics allows students to build a conceptual mathematical base and/or also illustrates mathematical practices and habits.

For all codes within this dimension, the aspect of instruction must be substantially correct to count as mid or high. Mid constitutes good use of many of these elements; high constitutes extraordinary use.

Note: All codes in this category are quality codes. A teacher can get a high score, even if the aspect of instruction you are scoring for occurs for only a portion of the segment.

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<b>Linking and Connections</b>		
<p>This code refers to teachers' and students' explicit linking and connections:</p> <ul style="list-style-type: none"> <li>• Among different representations of mathematical ideas or procedures (e.g., a linear graph and a table both capturing a linear relationship)</li> <li>• Among different mathematical ideas (e.g., proportionality and linearity; fractions and ratios, etc.)</li> <li>• Across representations and mathematical ideas/procedures (e.g., discussing how linearity is captured in any of the following: a graph, a table, or a mathematical equation)</li> </ul> <p>Note: If links are made but underlying representation/idea is incorrect, do NOT count as linking and connections.</p>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>No linking and connections occur. Also score as low when connections are <b>completely</b> pro forma (e.g., "Yesterday we did adding fractions with like denominators, today we will do subtracting fractions with like denominators.").</p>	<p>Links and connections are present, but do not have the features included in high, or feature these only momentarily (e.g., "You can compare ratios the same way you compare fractions" or "You can see each step in the computation in this array model here.").</p>	<p>Links and connections are present with sustained, careful work characterized by one of the following features:</p> <ul style="list-style-type: none"> <li>• Explicitness about how two or more ideas, procedures, or representations are related (e.g., pointing to specific areas of correspondence) OR</li> <li>• Detail and elaboration about how two mathematical ideas, procedures, or representations are related to one another (e.g., providing information about under what conditions the relationship occurs; noting meta-features; discussing implications of relationship)</li> </ul>

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## Mathematical Quality of Instruction (MQI)

<b>Explanations</b>		
<p>Mathematical explanations focus on why. This includes giving mathematical meaning to ideas or procedures, meaning of steps, or solution methods, e.g.:</p> <ul style="list-style-type: none"> <li>• Why a procedure works</li> <li>• Why a solution method makes sense</li> <li>• Why an answer is true</li> <li>• What a solution means in the context of the problem</li> </ul> <p>Examples: the reason for steps in simplifying fractions (dividing by <math>\frac{2}{2}</math>, for example, is same as dividing by 1; anything divided by 1 is still itself); why particular steps in a complex problem are justified or work to achieve the solution; that in a division problem, an answer of 9 R2 means each child gets 9 cookies and there will be 2 cookies left over.</p> <p>Do NOT count “how” e.g., descriptions of steps here (first I did x, then I did y) or simply providing definitions unless meaning is also attached.</p> <p>Note: Do NOT count incorrect or incomplete explanations as explanations.</p>		
Low	Mid	High
<p>No mathematical explanations are offered by the teacher or students or the “explanations” provided are simply descriptions of steps of a procedure.</p>	<p>The explanations offered by the teacher or students meet <i>any</i> of the following criteria:</p> <ul style="list-style-type: none"> <li>• They meet criteria under high, but they are not detailed</li> </ul> <p style="text-align: center;">OR</p> <ul style="list-style-type: none"> <li>• They are not general (they pertain only to the specific task/problem under consideration)</li> </ul> <p style="text-align: center;">OR</p> <ul style="list-style-type: none"> <li>• They are definitions that contain elements of “why” but that do not fully explain</li> </ul>	<p>The explanations offered by the teacher or students meets the following criterion:</p> <ul style="list-style-type: none"> <li>• They move beyond particular problems (i.e., they are general explanations, not for particular problems)</li> </ul> <p>AND meets <i>one or more</i> of the following criteria:</p> <ul style="list-style-type: none"> <li>• They give meaning to steps, procedures and solutions</li> <li>• They are detailed</li> </ul>

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<b>Multiple Procedures or Solution Methods</b>		
<p>Multiple procedures or solution methods occur in the segment:</p> <ul style="list-style-type: none"> <li>• Multiple solution methods for a single problem (including shortcuts)</li> <li>• Multiple procedures for a given problem type</li> </ul> <p>Defined as, e.g.:</p> <ul style="list-style-type: none"> <li>• Taking different mathematical approaches to solving a problem (e.g., comparing fractions by finding a common denominator AND comparing fractions by finding a common numerator)</li> <li>• The teacher or students solve a (word) problem using two different strategies.</li> </ul> <p>If the initial strategy(ies) occurred in a prior segment, score the second (and/or subsequent) segment at this code (i.e., no need to go back and score the initial segment).</p> <p>Note: Here you could also include instances when the teacher or a student mentions multiple different procedures or solution methods even if only one of them is enacted.</p> <p>Note: Do NOT count incorrect procedures or solution methods.</p>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>No evidence of multiple procedures or solution methods for single problem or a given problem type.</p>	<p>Multiple procedures or solution methods present, but do not have the features included in high, or feature these only momentarily (e.g., “this method is easier than the other” without explicit discussion of why).</p>	<p>Multiple procedures or solution methods occur at some length and with special features:</p> <ul style="list-style-type: none"> <li>• Explicit comparison of multiple procedures or solution methods for efficiency, appropriateness, ease of use, or other advantages and disadvantages</li> <li>• Explicit discussion of features of a problem that cues the selection of a particular procedure</li> <li>• Explicit links between multiple procedures or solution methods (e.g., how one is like or unlike the other)</li> </ul>

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**Developing Mathematical Generalizations**

Teacher and/or students **develop** mathematical generalizations by examining instances or examples, then making a general statement (e.g., drawing functions  $x^2$ ,  $2x^2$ ,  $4x^2$  then making a generalization about shape as coefficient changes).

Examples of this activity include:

- Examining particular cases and noticing a pattern
- Saying whether mathematical procedures work in all cases
- “Building up” a mathematical definition or deriving a mathematical property (e.g., defining “polygons” after considering different examples and non-examples of polygons)

Notes:

1. Requires at least two examples (either explicitly worked or referred to) from which generalization emerges.
2. Score generalizations for only the segment in which the generalization emerges/becomes explicit.
3. Do NOT count incorrect generalizations.
4. Do NOT count when teachers *state* generalizations without first developing them from examples.

Low	Mid	High
No generalizations are developed in this segment.  Also score as low for non-mathematical generalizations (e.g., drawing a picture helps solve a word problem).	Students/teacher <i>develop</i> a generalization, but the generalization developed is not complete, clear or detailed.	Teacher and/or students <i>develop</i> a generalization. The generalization should contain the <i>mathematical</i> essence of the work done with regards to a particular task and should be complete, clear and detailed.

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## Mathematical Quality of Instruction (MQI)

<b>Mathematical Language</b>		
<p>This code captures how fluently the teacher (and students) use mathematical language and whether the teacher supports students' use of mathematical language.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Fluent use of technical language</li> <li>• Explicitness about mathematical terminology</li> <li>• Encouraging students to use mathematical terms</li> </ul>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>Teacher does not demonstrate fluency in mathematical language. Teacher uses non-mathematical terms to describe mathematical ideas and procedures AND/OR teacher talk is characterized by sloppy/incorrect use of mathematical terms.</p> <p>If there is little mathematical language used, score as low.</p>	<p>Teacher uses mathematical language as a vehicle for conveying content, but has few or none of the special features listed under high. This is the default score when teacher is using mathematical language neither sloppily nor outstandingly.</p> <p>Also score as mid when segment has both features of high but includes some linguistic sloppiness.</p>	<p>Teacher uses mathematical language correctly and <i>fluently</i>.</p> <p>May include explicitness about terminology, reminding students of meaning, pressing students for accurate use of terms, encouraging student use of mathematical language. Density of mathematical language is high during periods of teacher talk.</p> <p>Dense, fluent, and accurate student talk can also count here.</p>

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## Mathematical Quality of Instruction (MQI)

<b>Overall Richness of the Mathematics</b>		
<p>This code captures the depth of the mathematics offered to students.</p> <p>Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of richness.</p>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>Components of richness are present but are incorrect OR Elements of rich mathematics are not present or are only minimally present.</p>	<p>Elements of rich mathematics are present but are used in a conventional way, without special features listed in high. May be characterized by some mid scores in the codes of this dimension, an isolated high along with substantial procedural focus, etc.</p> <p>Can also be used for combination of correct and incorrect rich elements.</p>	<p>Elements of rich mathematics are present, And either:</p> <p>a) there is truly outstanding performance in one or more of the elements (even for a brief portion of the segment)</p> <p style="text-align: center;">OR</p> <p>b) there is a combination of elements that either saturate the segment with mathematical meaning or foster student proficiency with mathematical practices.</p> <p>For example:</p> <ul style="list-style-type: none"> <li>• Focus is on meaning via representations linked to one another or to underlying ideas and/or explanations that generalize;</li> <li>• Focus is on explicit comparing of solution methods or procedures (e.g., most efficient) and/or generalizations that are developed from specific examples; a focus on meaning not necessary during this discussion.</li> </ul>

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## Working with Students and Mathematics

This dimension captures whether teachers can understand and respond to students' mathematically substantive productions (utterances or written work) or mathematical errors. By mathematically substantive productions, we mean questions, claims, explanations, solution methods, ideas, etc. that contain substantial mathematical ideas. By students' mathematical errors, we mean those incorrect student productions that offer opportunities for discussing and addressing pertinent mathematical ideas.

### Remediation of Student Errors and Difficulties

With this code, we mean to mark instances of remediation in which student misconceptions and difficulties with the content are substantially addressed.

Note: Can occur during active instruction or small group/partner/individual work time.

Note: Remediation must have mathematical content.

Low	Mid	High
<p>No conceptual remediation occurs for any of the following reasons:</p> <ul style="list-style-type: none"> <li>• There are no student misunderstandings or difficulties with the content</li> <li>• Remediation is procedural and brief or otherwise non-substantive</li> <li>• The teacher chooses not to remediate</li> <li>• The teacher remediation is confusing or off-track</li> </ul>	<p>Teacher engages in conceptual remediation <i>briefly or occasionally</i>.</p> <p>Mid also includes very <i>extensive</i> procedural remediation.</p>	<p>Teacher engages in conceptual remediation <i>systematically and at length</i>.</p> <ul style="list-style-type: none"> <li>• Identifying the source of student errors or misconceptions</li> <li>• Discussing how student errors illustrate broader misunderstanding and then addressing those errors</li> </ul> <p>Also score high for any instance, however brief, of the following:</p> <ul style="list-style-type: none"> <li>• Anticipating common student errors and providing instruction that helps avoid error</li> <li>• Parsing student productions to separate correct and incorrect thinking</li> </ul>

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## Mathematical Quality of Instruction (MQI)

<b>Responding to Student Mathematical Productions in Instruction</b>		
<p>Student contributes substantive mathematical production(s) for the class. By mathematically substantive productions, we mean, for example, “why” questions, claims, explanations, generalizations, ideas, or a complex description of a solution method, etc. We do not mean simply answers to problems or pointed questions where teacher has sought a specific, bounded piece of information. Productions tend to be student explanation, questioning, and/or reasoning.</p> <p>Teacher understands and responds to student productions during instruction in mathematically appropriate ways such as:</p> <ul style="list-style-type: none"> <li>• Identifying mathematical insight in specific student questions, comments, or work</li> <li>• Building instruction on student ideas or methods</li> </ul>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>Routine instruction with no student productions.</p> <p style="text-align: center;">OR</p> <p>There are student productions, but no evidence that student ideas are “heard,” taken up, or used in instruction. Teacher may provide evidence that he/she does not understand student productions.</p> <p style="text-align: center;">OR</p> <p>Teacher uses student productions but in a way that muddles or confuses the mathematics of the lesson.</p>	<p>Student productions are present.</p> <p style="text-align: center;">AND</p> <p>Teacher may engage in features listed under high briefly, but instruction generally proceeds <i>without</i> strong use of student mathematical ideas.</p> <p style="text-align: center;">OR</p> <p>There is evidence that the teacher understands student thinking but chooses not to use it at that time.</p>	<p>Student productions are present.</p> <p style="text-align: center;">AND</p> <p>Teacher “hears” what students are saying, mathematically, and responds appropriately during instruction. Students’ mathematical ideas are woven <i>at length</i> into the <i>development</i> of mathematical ideas during the lesson.</p> <p>In particular, teacher may comment on student’s mathematical ideas, elicit further student clarification of ideas, ask other students to comment on ideas, expand on and reinforce student utterances, etc.</p> <p>Other markers include:</p> <ul style="list-style-type: none"> <li>• Identifying key ideas in student statement (“Mark had an interesting idea...”)</li> <li>• Highlighting key features of student questions (“Do you see Mark asked a question about whether this would work in all cases?”)</li> <li>• Identifying a student with an idea (“Mark’s method”)</li> </ul>

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## Mathematical Quality of Instruction (MQI)

<b>Overall Working with Students and Mathematics</b>		
<p>Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the teachers' interactions with the students around the content.</p>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>Few substantive interactions between teacher and students. Errors may occur but teacher addresses briefly/procedurally.</p> <p style="text-align: center;">OR</p> <p>Substantive student mathematical productions or errors do occur, but teacher does not respond to or use those productions</p> <p style="text-align: center;">OR</p> <p>Teacher responses to student productions lead the lesson off-track</p>	<p>A combination of strong and weak features (e.g., some high-level remediation but teacher ignores students' contributions; extended and detailed/organized procedural remediation but no student productions).</p>	<p>Strong and significant teacher understanding and use of student ideas and errors around the content as evident by <i>outstanding</i> performance in one area or solid performance in both.</p>

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<b>Errors and Imprecision</b>		
<p>This category is intended to capture teacher errors or imprecision in language and notation, uncorrected student errors, or the lack of clarity/precision in the teacher's presentation of the content.</p> <p>Do NOT count errors that are noticed and corrected within the segment.</p>		
<b>Major Mathematical Errors</b>		
<ul style="list-style-type: none"> <li>• Solving problems incorrectly</li> <li>• Defining terms incorrectly</li> <li>• Forgetting a key condition in a definition</li> <li>• Equating two non-identical mathematical terms, etc.</li> </ul>		
Low	Mid	High
<p>Instruction is <i>clean</i> of major errors in spoken or written work OR errors that occur are noticed and corrected within the segment.</p>	<p>Teacher makes <i>major errors</i> either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem).</p> <p>The errors occur in <i>part</i> of the segment.</p>	<p>Teacher makes <i>major errors</i> either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem).</p> <p>The errors occur in <i>most</i> of the segment.</p>

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Imprecision in Language or Notation (Mathematical Symbols)		
<ul style="list-style-type: none"> <li>• Errors in notation (mathematical symbols)</li> <li>• Errors in mathematical language</li> <li>• Errors in general language</li> </ul> <p><b>Definitions</b></p> <ul style="list-style-type: none"> <li>• <b>Notation</b> includes conventional mathematical symbols, such as +, -, =, or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents. Errors in notation might include inaccurate use of the equals sign, parentheses, or division symbol. By “conventional notation,” we do not mean use of numerals or mathematical terms.</li> <li>• <b>Mathematical language</b> includes technical mathematical terms, such as “angle,” “equation,” “perimeter,” and “capacity.” If a teacher uses these terms incorrectly, record as an error. When the focus is on a particular term or definition, also score errors in spelling or grammar.</li> <li>• Teachers often use “<b>general language</b>” to convey mathematical concepts (i.e., explaining mathematical ideas or procedures in non-technical terms). General language also includes analogies, metaphors, and stories. Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. If teacher is unclear in his/her general talk about mathematical ideas, terms, concepts, procedures, record as an error.</li> </ul>		
Low	Mid	High
Instruction is <i>clean</i> of errors in mathematical language, general language, and notation. Any errors made and quickly corrected should also be scored here.	Teacher makes a <i>small number</i> of momentary errors in notation, mathematical or general language.	Instruction is characterized of linguistic and notational <i>sloppiness across the segment</i> and/or by <i>major</i> notational and linguistic errors in even a small number of mathematical terms.

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## Mathematical Quality of Instruction (MQI)

<b>Lack of Clarity in Presentation of Mathematical Content</b>		
<ul style="list-style-type: none"> <li>• Teacher utterances cannot be understood, e.g.:                             <ul style="list-style-type: none"> <li>• Mathematical point is muddled, confusing, or distorted</li> <li>• Language or major errors make it difficult to discern the point</li> <li>• Teacher neglects to clearly solve the problem or explain content</li> </ul> </li> <li>• Teacher’s launch of a task/activity lacks clarity (the “launch” is the teacher’s effort to get the mathematical tasks/activities into play)</li> </ul>		
Low	Mid	High
Teacher’s presentation of the mathematical content and/or launching of tasks is <i>clear</i> and <i>unambiguous</i> .	Teacher’s presentation of the content and/or launching of tasks is <i>not clear</i> for <i>portions</i> of the segment.	Teacher’s presentation of the mathematical content is <i>unclear</i> , <i>vague</i> , or <i>incomplete</i> for most of the segment.  OR Teacher’s work is <i>muddled</i> or <i>confusing</i> and <i>severely distorts</i> the mathematical essence of the content.  Also, teacher conveys mathematical tasks or problems <i>incompletely</i> or in a <i>confusing</i> manner.
<b>Overall Errors and Imprecision</b>		
Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the errors and imprecision in instruction.		
Low	Mid	High
No errors occur. Do not score as low if mid or high is marked in any category above.	Brief error or errors generally not serious enough to indicate teacher may lack mathematical knowledge.	Either multiple small errors, consistent lack of clarity, or one large error to suggest that teacher may lack key mathematical knowledge.

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## **Student Participation in Meaning-Making and Reasoning**

This dimension attempts to capture evidence of students' involvement in tasks that ask them to "do" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. During active instructional segments, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. During small group/partner/individual work times, this mainly occurs through work on a non-routine task.

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<b>Students Provide Explanations</b>		
<p>Students provide a mathematical explanation for an idea, procedure, or solution.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Students explain why a procedure works</li> <li>• Students explain the procedure they used to solve a particular problem by attending to the meaning of the steps involved in this procedure rather than simply listing those steps</li> <li>• Students explain what an answer means</li> <li>• Students explain why a solution method is suitable or better than another method</li> <li>• Students explain an answer based on an estimate or other number-sense reasoning</li> </ul> <p>Notes:</p> <ul style="list-style-type: none"> <li>• Explanations could be initiated by the teacher or self-initiated; they could also be co-constructed with the teacher or constructed individually</li> <li>• Explanations do not have to be complete or correct</li> </ul>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>No student explanations are featured in the segment.</p>	<p>The explanations offered:</p> <ul style="list-style-type: none"> <li>• Pertain to a specific problem/task</li> <li>• Do not generalize to key mathematical ideas of the content under consideration</li> </ul> <p>Examples:</p> <ul style="list-style-type: none"> <li>• A student explains that <math>\frac{3}{4}</math> is larger than <math>\frac{3}{5}</math> because the denominator in the first fraction is smaller than the denominator in the second fraction.</li> <li>• In the set 4, 6, 7, 10, 11 the 7 is the median because it has two data points on either side.</li> </ul>	<p>The explanations offered generalize past specific problems to address key mathematical ideas of the content under consideration.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• A student explains that <math>\frac{3}{4}</math> is larger than <math>\frac{3}{5}</math> because the denominator in the first fraction is smaller than the denominator in the second fraction and fractions with smaller denominators correspond to larger pieces.</li> <li>• In any set with an odd number of data points, the median will be the middle data point because there will be an equal number of data points on either side.</li> </ul>

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**Student Mathematical Questioning and Reasoning**

Students engage in mathematical questioning or reasoning, including:

- Students provide counter-claims in response to a proposed mathematical statement or idea (whether from another student, the teacher, or a text)
- Students ask mathematically motivated questions requesting explanations (e.g., “Why does this rule work?” “What happens if all the numbers are negative?”)
- Students provide examples of a phenomena
- Students make conjectures about the mathematics discussed in the lesson (e.g., “I’ve been trying to make a triangle with two obtuse angles and I don’t think you can.”)
- Students form conclusions based on patterns they identify or on other forms of evidence (e.g., “Because the sum of the angles of any triangle is 180 degrees, a triangle cannot have two obtuse angles.”)
- Students engage in reasoning about a hypothetical or general case (e.g., “Because the sum of the angles of any triangle is 180 degrees, a triangle should have at least two acute angle.”)

Note: Students’ productions do not have to be complete or correct.

Low	Mid	High
The segment does <i>not</i> feature any of the student behaviors captured by this code.	The segment features <i>one or two</i> of the student behaviors captured by this code.	The segment features <i>three or more</i> of the student behaviors captured by this code.

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<b>Enacted Task Cognitive Activation</b>		
<p>This code refers to the <u>enactment</u> of the task, regardless of the initial demand of the curriculum/textbook task or how the teacher sets up the task for students.</p> <p>Notes:</p> <ul style="list-style-type: none"> <li>• Student confusion does not necessarily suggest that students are engaging with the content at a high cognitive level.</li> <li>• Working on review tasks or on ideas discussed in previous lessons does not necessarily mean that students use lower order thinking skills.</li> <li>• This code should not be confounded with the difficulty of the task or whether it is appropriate for a certain grade-level.</li> </ul>		
<b>Low</b>	<b>Mid</b>	<b>High</b>
<p>Students engage with the content at a <i>low</i> cognitive level.</p> <p>Examples of cognitively <i>undemanding</i> activities include:</p> <ul style="list-style-type: none"> <li>• Recalling and applying well-established procedures</li> <li>• Recalling or reproducing known facts, rules, or formulas</li> <li>• Listening to a teacher presentation with limited student input</li> <li>• Going over homework with little additional student work (e.g., reporting numerical answers)</li> <li>• Unsystematic exploration (i.e., students do not make <i>systematic and sustained progress in developing mathematical strategies or understanding</i>)</li> </ul>	<p>Students engage with content at a <i>mixed level</i> of cognitive activation. May also include:</p> <ul style="list-style-type: none"> <li>• Tasks with variable enactment (high and then low during segment)</li> <li>• Direct instruction with substantive student input at certain points</li> <li>• Tasks with middling cognitive activation</li> </ul>	<p>Students engage with content at a <i>high</i> level of cognitive activation.</p> <p>Examples of cognitively <i>activating</i> activities include when students:</p> <ul style="list-style-type: none"> <li>• Determine the meaning of mathematical concepts, processes, or relationships</li> <li>• Draw connections among different representations or concepts</li> <li>• Make and test conjectures</li> <li>• Look for patterns</li> <li>• Examine constraints</li> <li>• Explain and justify</li> </ul>

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<b>Overall Student Participation in Meaning-Making and Reasoning</b>		
<p>This code attempts to capture evidence of students' involvement in "doing" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning.</p> <ul style="list-style-type: none"> <li>• During <b>active instruction segments</b>, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking.</li> <li>• During <b>small group/partner/individual work time</b>, this mainly occurs through work on a non-routine task.</li> </ul> <p>Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the student participation in meaning-making and reasoning.</p>		
Low	Mid	High
<p>There are only a few or no examples of student participation in meaning making and reasoning. Tasks are largely procedural in nature. Also score as low if there are unproductive explorations in which <i>the majority</i> of the students are off-track, mathematically.</p>	<p>Students engage with content at <i>mixed level</i>. Students may provide substantive explanations or ask mathematically motivated questions. May also include tasks with variable enactment (high and then low during segment).</p>	<p>Students contribute substantially or engage productively in activities that can lead to meaning-making and reasoning. Such contributions are a major feature of the segment, with many student contributions, or extended work on a challenging task.</p>

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## OVERALL LESSON CODES

<b>Overall MQI and MKT</b>		
<p>These codes are intended to capture the overall mathematical quality of instruction (MQI) and the teacher’s mathematical knowledge for teaching (MKT) as suggested by the teacher’s work during the lesson.</p>		
<b>Whole-Lesson Mathematical Quality of Instruction (MQI)</b>		
Low	Mid	High
<p>Instruction is characterized by combinations of the following:</p> <ul style="list-style-type: none"> <li>• Systematic teacher errors (mathematical errors, in notation, in language) or lack of clarity around the mathematics</li> <li>• Major teacher conceptual error in a significant portion of the lesson</li> <li>• Unproductive teacher-student interactions around the content (e.g., teacher cannot effectively remediate)</li> <li>• Lack of directionality/ unsystematic exploration</li> <li>• Lack of connection of classroom activities to mathematical content</li> </ul>	<p>Instruction does not have characteristics of low and is mostly error-free, but lacks the mathematical richness, appropriate use and discussion of procedures, and the sharp mathematical focus detailed under high.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Mostly error-free procedural instruction, perhaps with occasional but not consistent elements of richness</li> <li>• Mainly pro forma interactions with students (inquiry, response, evaluation-type discussion)</li> </ul>	<p>Instruction is error-free (save for MINOR slips) and characterized by combinations of the following:</p> <ul style="list-style-type: none"> <li>• Mathematical richness in terms of explanations, links, and connections</li> <li>• Focus on mathematical practices (developed generalizations, mathematical efficiency) that is sustained and detailed</li> <li>• Instruction has a clear and sharp mathematical focus and directionality that allows students to develop the important mathematical ideas under consideration</li> <li>• Instruction is characterized by at least some productive teacher-student interactions around the content (either working with student ideas/errors OR student participation in mathematical meaning-making)</li> </ul>

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## Mathematical Quality of Instruction (MQI)

<b>Lesson-Based Guess at Mathematical Knowledge for Teaching (MKT)</b>		
<p>How do you think the teacher would score on our MKT assessment?</p> <p>Note: Differs from whole-lesson MQI in that raters may use more judgment to estimate generally what teacher MKT could be. For instance, lesson captured may be mostly practice, but there may be some evidence of very strong teacher MKT.</p>		
Low	Mid	High
<p>MKT is estimated as low, as suggested by combinations of the following:</p> <ul style="list-style-type: none"> <li>• Inappropriate use of representations, mathematical notation and language</li> <li>• Mathematical errors</li> <li>• Inappropriate explanations, descriptions of procedures, or discussion of student ideas and contributions</li> <li>• Teacher puzzlement and incorrect or distorted presentation of the content</li> </ul>	<p>MKT is estimated as mid as suggested by the following:</p> <ul style="list-style-type: none"> <li>• Instruction is mostly error-free but there is no evidence teacher has ability to provide rich instruction, understands lesson material deeply, or has capacity to work with students' thinking or errors</li> </ul>	<p>MKT is estimated as high, as suggested by combinations of the following:</p> <ul style="list-style-type: none"> <li>• Ability to provide accurate and rich instruction</li> <li>• Strong use of mathematical language</li> <li>• Ability to follow and build on mathematical ideas</li> <li>• Ability to unpack the content and make it accessible to students</li> <li>• Ability to identify and remediate student errors and misconceptions</li> </ul>

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