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Omar Khayyam: Much More Than a Poet

by

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Omar Khayyam was born in 1048 A.D. in Naishapur, Khorrasan (Iran), the same city where he later died. His full name was Ghiyath al-Din Abu'l-Fath Umar ibn Ibrahim Al-Nisaburi-Khayyami. Some believe that he was descended of the Arab tribe of Khayyami. In addition to being educated in his hometown of Naishapur, Omar Khayyam studied in Bukhara, Balkh, Samarqand and Isphahan. Most of his life, however, was spent in Naishapur and Samarqand.

According to a biography of Omar Khayyam by Edward J. Fitzgerald, "His Takhallus or poetical name (Khayyam) signifies a Tent-maker, and he is said to at one time exercised that trade, ... Many Persian poets similarly derive their names from their occupations." Khayyam himself wrote:

Khayyam, who stitched the tents of science,
Has fallen in grief's furnace and been suddenly burned,
The shears of Fate have cut the tent ropes of his life,
And the broker of Hope has sold him for nothing!

Omar Khayyam, although well known for his poetry, was also an accomplished mathematician, scientist, astronomer, and philosopher. In fact, his contributions include the Jalali Calendar, astronomical tables, and contributions to mathematics, especially in Algebra. He wrote, "Maqalat fi al-Jabr al-Muqabila," in this area of mathematics, which many claim provided great advancement in the field. Dr. Zahoor begins to describe some of Omar Al-Khayyam's feats stating in his research, "He classified many algebraic equations based on their complexity and recognized fourteen different forms of cubic equations. Omar Khayyam developed a geometrical approach to solving equations, which involved an ingenious selection of proper conics. He solved cubic equations by intersecting a parabola with a circle. Omar Khayyam was the first to develop the

binomial theorem and determine binomial coefficients. He developed the binomial expansion for the case when the exponent is a positive integer... He extended Euclid's work giving a new definition of ratios and included the multiplication of ratios. He contributed to the theory of parallel lines."

His work in algebra was highly valued throughout Europe in the Middle Ages. However, he is best known for his poetic work, "Rubaiyat" (quatrains), which was translated by Edward Fitzgerald in 1859. In his work, "Rubaiyat of Omar Khayyam," Mr. Fitzgerald refers to him as "The Astronomer-Poet of Persia." Mr. Fitzgerald goes on to quote from the Appendix to "Hyde's Veterum Persarum Religio," reciting, "It is written in the chronicles of the ancients that this King of the Wise, Omar Khayyam, died at Naishapur in the year of Hegira, 517 (A.D. 1123); in science he was unrivaled, ... the very paragon of his age." In an article by J.J. O'Connor and E.F. Robinson of the School of Mathematics and Statistics, University of St. Andrews, Scotland, they attribute the lack of common knowledge of Al-Khayyam's scientific achievements to popularity of Mr. Fitzgerald's translation of the 'Rubaiyat,' stating Khayyam's fame as a poet has caused some to forget about his scientific achievements which were much more substantial." They go on to add, "Versions of the forms and verses existed in Persian literature before Khayyam, and only about 120 of the verses can be attributed to him with certainty."

Omar Khayyam also provided insight in other areas of science. He wrote two books in metaphysics, "Risala Dar Wujud" and "Nauruz Namah." Of an unknown total number of works, ten books and thirty monographs have survived including four books on mathematics, one on algebra, one on geometry, three on physics, and two books on metaphysics (Zahoor p2).

The political events of the 11th century played a major role in the course of Khayyam's life and he was impacted greatly by the unrest in the region. The Seljuq Turks invaded southwestern Asia in the eleventh century and ultimately acquired Mesopotamia, Syria, Palestine, and most of Iran. Between 1038 and 1040, they subjugated all of northeastern

Iraq. The Seljuq ruler Toghril Beg proclaimed himself sultan at Naishapur in 1038 (O'Connor & Robinson p2).

In 1070 Omar Khayyam moved to Samarkand in Uzbekistan. There Khayyam was supported by Abu Tahir, a "prominent jurist" of Samarkand, and this allowed him to write his most famous algebra work, "Maqalat fi al-Jabr al-Muqabila" ("Treatise on Demonstration of Problems of Algebra"), which contained a complete classification of cubic equations with geometric solutions found by means of intersecting conic sections. Khayyam left Esfahan and went to Merv (now Mary, Turkmenistan).

Toghril Beg made Esfahan the capital of his domains and his grandson, Malik-Shah Jalal al-Din, was the ruler of that city from 1073. Omar Khayyam was invited by Malik-Shah to go to Esfahan to set up an Observatory there. Khayyam led a group of scientists there for eighteen years giving him much time to devote to his work (O'Connor & Robertson p2).

In addition to compiling astronomical tables, he also contributed to calendar reform in 1079. Malik-Shah had asked for, "An accurate calendar to be used for revenue collections and various administrative matters." Dr. Zahoor also points out, "His calendar, 'Al-Tarikh-al-Jalali,' is superior to the Gregorian calendar and is accurate to within one day in 3770 years. Specifically, he measured the length of the year as 365.24219858156 days. It shows that he recognized the importance of accuracy by giving his result to eleven decimal places. As a comparison, the length of the year in our time is 365.242190 days. This number changes slightly in the sixth decimal place, e.g., in the nineteenth century it was 365.242196 days." The result was the Jalali era, named from Jalal-ud-din one of the new sultan's names.

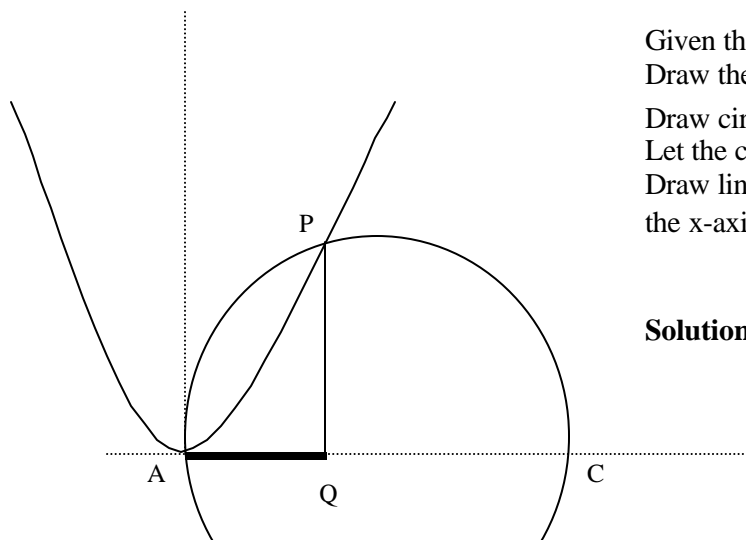
In 1092, again politics became a burden upon Khayyam's work. Malik-Shah died late in the year and his second wife took over as ruler for two years. She had not gotten along with her deceased husband's advisor, Nizam al-Mulk; therefore, those that he had supported in Malik-Shah's rule were no longer receiving that support. Both the

Observatory and Khayyam's calendar reform were abandoned for lack of funding (O'Connor & Robertson p2).

Sometime later, Malik-Shah 's third son, Sanjar, became the new ruler of the Seljuq Empire. He created a learning institution where Kahayyam was to continue his work in mathematics. In one of Kahayyam's mathematics papers discussed by O'Connor and Robertson, they refer to his writing, "Find a point on a quadrant of a circle in such a manner that when a normal is dropped from the point to one of the bounding radii, the ratio of the normal's length to that of the radius equals the ratio of the segments determined by the foot of the normal." They state that, "Khayyam shows that this problem is equivalent to solving a second problem, 'Find a right triangle having the property that the hypotenuse equals the sum of one leg plus the altitude on the hypotenuse.'"

Khayyam was then led to solve the following cubic, $x^3 + 200x = 20x^2 + 2000$ finding a positive root by considering the intersection of a rectangular hyperbola and a circle.

KHAYYAM'S CUBIC SOLUTION



Given the cubic $x^3 + ax^2 = b$.
 Draw the parabola $x = ay$
 Draw circle diameter $AC = b/a^2$ on x-axis
 Let the circle and parabola meet at P
 Draw line from P perpendicular to the x-axis meeting it at Q.

Solution to the cubic is AQ

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"An approximate numerical solution was then found by interpolation in trigonometric tables" (O'Connor & Robertson p4).

Khayyam made it evident that he was not finished, and that he planned to explain in more detail solutions to cubic equations when he wrote, "If the opportunity arises and I can succeed, I shall give all these fourteen forms with all their branches and cases, and how to distinguish whatever is possible or impossible so that a paper containing elements which are greatly useful in this art will be prepared" (O'Connor & Robertson)

O'Connor and Robertson came to the conclusion that "Khayyam himself seems to have been the first to conceive a general theory of cubic equations. Khayyam wrote, 'In the science of algebra one encounters problems dependant on certain types of extremely difficult preliminary theorems, whose solution was unsuccessful for most of those who attempted it. As for the Ancients, no work from them dealing with the subject has come down to us; perhaps after having looked for solutions and having examined them, they were unable to fathom their difficulties; or perhaps their investigations did not require such an examination; or finally, their works on this subject, if they existed, have not been translated into our language.'" O'Connor and Robertson also claim that, "another major achievement in the algebra text is Khayyam's realisation that a cubic equation can have more than one solution. He demonstrated the existence of equations having two solutions, but unfortunately he does not appear to have found that a cubic can have three solutions."

As discussed by O'Connor and Robertson, "Khayyam used a method of finding n th roots based on the binomial expansion, and therefore on the binomial coefficients." Khayyam himself wrote, "The Indians possess methods for finding the sides of squares and cubes based on such knowledge of the squares of nine figures, that is, the square of 1,2,3, etc., and also the products formed by multiplying them by each other, i.e. the products of 2,3 etc. I have composed a work to demonstrate the accuracy of these methods, and have proved that they do lead to the sought aim. I have moreover increased the species, that is, I have shown how to find the sides of the square-square, quatro-cube, cubo-cube, etc., to any length, which has not been made before now. The proofs I gave on this occasion are only arithmetic proofs based on the arithmetical parts of Euclid's 'Elements.'"

In discussing Omar Khayyam's geometric approach to cubic equations, Victor Katz wrote, "Of course, this approach had been used earlier by Menaechmus and others to solve certain special cubics (notably in relation to the problem of "duplicating the cube"), but Khayyam generalized it to cover essentially all cubics (albeit with many individual cases so as to avoid negative numbers). It's usually said that Khayyam erroneously believed the cubic could not be solved algebraically, but I think we need to be careful about assuming that Khayyam was referring to the modern idea of what constitutes an 'algebraic' solution." Mr. Katz supports his beliefs by citing Khayyam, writing that, "... no attention should be paid to the fact that algebra and geometry are different in appearance. Algebras are geometric facts which are proved." Katz goes on to explain that Khayyam, through this, "contributed to reconciling the two fields of geometry and algebra... thereby casting Khayyam as a forerunner of Descartes. Khayyam was definitely far more inclined than the Greeks to treat his geometrical line segments as numerical quantities rather than strictly as spatial agnitudes."

Khayyam also made a contribution to "non-euclidean" geometry. "In trying to prove the parallels postulate, he accidentally proved properties of figures in non-euclidean geometries. Khayyam also gave important results on ratios in this book, extending Euclid's work to include the multiplication of ratios. The importance of Khayyam's contribution is that he examined both Euclid's definition of equality of ratios (which was first proposed by Eudoxus) and the definition of equality of ratios as proposed by earlier Islamic mathematicians such as al-Mahani which was based on continued fractions. Khayyam proved that the two definitions are equivalent (O'Connor & Robertson p4).

Katz again notes Khayyam in his writing, "This cannot be solved by plane geometry, since it has a cube in it. For the solution we need conic sections." He goes on to credit Khayyam with "anticipating the eventual proof of the insolvability of the Delian problem," or duplicating the cube, leading us to believe that he thinks that this may help to explain Khayyam's statement that cubics cannot be "solved algebraically."

Katz reminds us that, "To Khayyam, 'algebras are geometric facts which are PROVED,' and he was still strongly influenced by the Greek insistence on straight-edge and compass constructions as the only valid 'proofs' in a certain strict formal sense." Katz then concludes that when Khayyam declared that the cubic could not be "solved algebraically" his definition of "an algebra" was geometrically "proved." Katz writes, "Of course, with this interpretation his statement was perfectly correct, and in fact, was simply another way of expressing his assertion that the Delian problem cannot be solved by straight-edge and compass."

Omar Khayyam died in 1123, but not before he was to write his wishes,

Ah, with the Grape my fading Life provide,
And Wash my Body whence the life has died,
And in a Windingsheet of Vine-leaf wrapt,
So bury me by some sweet Garden-side. (LXVII of Rubaiyat)

In recounting a personal story of a pupil of Khayyam's, Mr. Fitzgerald quotes, "Years after, when I chanced to revisit Naishapur, I went to his final resting place, and lo! It was just outside a garden, and trees laden with fruit stretched their boughs over the garden wall, and dropped their flowers upon his tomb, so that the stone was hidden under them."

Looking at just a few of the accomplishments of Omar Khayyam one cannot help but be astonished. His mind was quite obviously ahead of the time. From creating the Al-Tarkh-al-Jalali, a calendar with unbelievable accuracy, to writing, "Maqalat fi al-Jabr al-Muqabila," a book on algebra that stated that cubics could not be solved by ruler and compass methods but require the use of conics, which would not be proven until seven hundred and fifty years later. A poet that excelled not only in prose, but whose ability in language flowed freely from written word to the language of mathematics and science.

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