Traverse Calculations

Traversing Methods

- Angle and Distance (Total Station Surveys)
- Direction and Distance (Compass-tape Surveys)

Traversing Methods

- Angle and Distance (Total Station Surveys)

Traversing Methods

- Direction and Distance (Compass-tape Surveys)

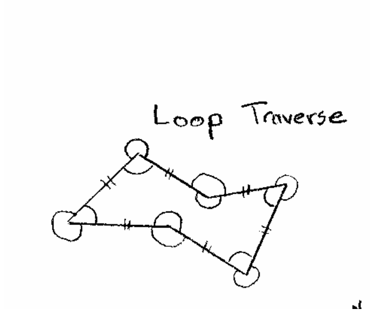
Traversing Methods

- Direction and Distance (Electronic equipment)

Traverse Types

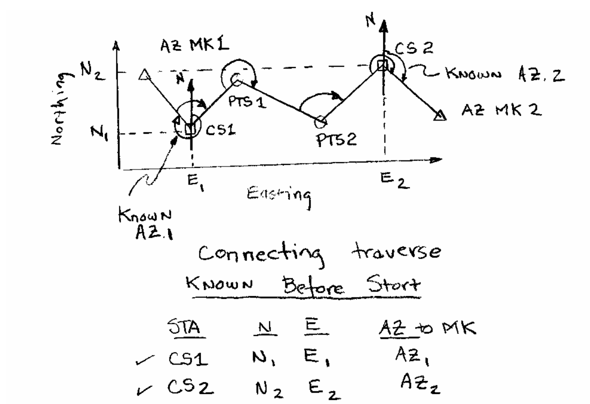
- Loop
  – Geometrically closed
  – Mathematically closed (checks possible)
- Connecting
  – Geometrically open
  – Mathematically closed (checks possible)
- Open
  – Geomet. and Math. open (no check)

Loop Traverse



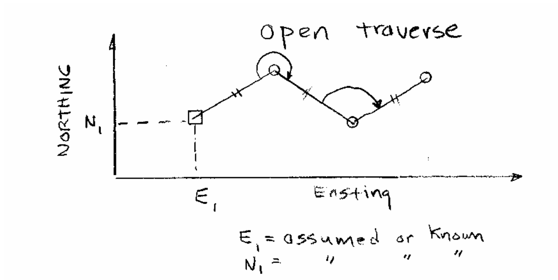
- Geometrically closed
- Mathematically closed (checks possible)

Connecting Traverse



- Geometrically open
- Mathematically closed (checks possible)

Open Traverse



Geometrically open
Mathematically open

Overview of Calculations

- Distance reduction
- Angle closure and angle adjustment
- Calculate direction for each side
- Calculate Latitude and Departure
- Calculate Linear error of closure (LEC) and Relative error of closure (REC)

Overview (cont.)

- Adjust latitudes and departures (traverse adjustment)
- Compute coordinates (Northings (y) and Eastings (x))
- Compute Area
- Inverse Problem - Given the coordinates of two points find, dir. and dist. between them

Distance Reduction

$$\mathbf{HD} = \mathbf{SD} \cdot \mathbf{Cos}(\alpha) = \mathbf{SD} \cdot \mathbf{Sin}(\mathbf{ZA})$$

Perform Angle Closure

- Closure = (meas. or calc.) – (fixed or known)

- Closure should meet standard of accuracy required for traverse specification being followed. See ALTA-ACSM specifications for boundary traverses.

Angle Closure (cont.)

- If the angle closure fails to meet the standard of accuracy, a gross blunder or an unaccounted for systematic error is suspected among the measurements.
- In either case the measurement with the blunder or the error should be isolated and replaced by remeasurement before continuing with the calculations.

American Congress on Surveying and Mapping
Minimum Angle, Distance and Closure Requirements for Survey Measurements
Which Control Land Boundaries for ALTA-ACSM Land Title Surveys
(Note 1)

Dir. Reading of Instrument (Note 2)	Instrument Reading Estimated (Note 3)	Number of Observations Per Station (Note 4)	Spread From Mean of D&R Not To Exceed (Note 5)	Angle Closure Where N=No. of Stations Not To Exceed	Linear Closure (Note 6)	Distance Measurement (Note 7)	Minimum Length of Measurements (Notes 8, 9, 10)
20" <1'> [10"]	5" <0.1'> N.A.	2 D&R	5" <0.1'> [5"]	10" $\sqrt{N}$	1:15,000	EDM or Doubletape with Steel tape	(8) 81m, (9) 153m, (10) 20 m

Note (1) All requirements of each class must be satisfied in order to qualify for that particular class of survey. The use of a more precise instrument does not change the other requirements, such as number of angles turned, etc.

Note (2) Instrument must have a direct reading of at least the amount specified (not an estimated reading), i.e.: 20" = Micrometer reading theodolite, <1'> = Scale reading theodolite, [10"] = Electronic reading theodolite.

Note (3) Instrument must have the capability of allowing an estimated reading below the direct reading to the specified reading.

Note (4) D & R means the Direct and Reverse positions of the instrument telescope, i.e., Urban Surveys require that two angles in the direct and two angles in the reverse position be measured and meaned.

Note (5) Any angle measured that exceeds the specified amount from the mean must be rejected and the set of angles re-measured.

Note (6) Ratio of closure after angles are balanced and closure calculated.

Note (7) All distance measurements must be made with a properly calibrated EDM or Steel tape, applying atmospheric, temperature, sag, tension, slope, scale factor and sea level corrections as necessary.

Note (8) EDM having an error of 5mm, independent of distance measured (Manufacturer's specifications).

Note (9) EDM having an error of 10mm, independent of distance measured (Manufacturer's specifications).

Note (10) Calibrated steel tape.

Perform Angle Adjustment

- Starting point is correction per angle.
- C_A= correction per angle = - Closure ÷ n

  n = number of angles
- The C_A should be an integer number. Use integer arithmetic to calculate it.

Angle Adjustment (cont.)

- Adjust for Horizon Closure or Calculate Mean Int.
- Mean Int. = Int. + C_A

Field Angles					
Sta	Int.	Ext.	Sum	Closure	C_A
1	069°49'30"	290°12'05"			
2	083°16'55"	276°43'20"			
3	026°53'35"	333°06'15"			

Angle Adjustment (cont.)

Field Angles					
Sta	Int.	Ext.	Sum	Clos	C_A
1	69° 49' 30"	290° 12'05"	360°01'35"	+ 95"	- 48"
2	83° 16' 55"	276°43'20"	360°00'15"	+ 15"	- 07"
3	26° 53' 35'	333°06'15"	359°59'50"	- 10"	+ 05"

C_A = - (+95") ÷ 2 = - 48"
C_A = - (+15") ÷ 2 = - 07"

Angle Adjustment (cont.)

• Mean interior = Int. + C_A

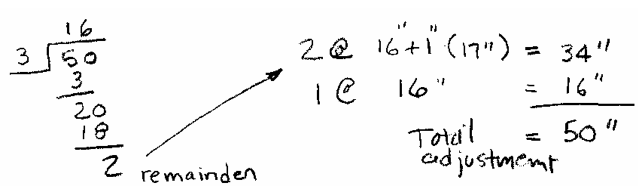
Sta.	field int.	C_A	Mean int.
1	69° 49’ 30”	- 48”	69°48’ 42”
2	83° 16’ 55”	-07”	83°16’ 48”
3	26° 53’ 35	+ 05”	26° 53’40”

Sum = 179°59’10”

Angle Adjustment (cont.)

• Calc. Adj. Int. Angle or adjust for geometric sum

• Closure = 179° 59’ 10” – 180° 00’ 00” = - 50”
• C_A = - (-50”) ÷ 3 (do division longhand, find remainder and adjust that many (remainder) angles by one second more ; prevents round off error)



Angle Adjustment (cont.)

Adjusted Interior = Mean int. + C_A

Sta.	Mean int.	C_A	Adj. int.
1	69°48’ 42”	+ 17”	69°48’ 59”
2	83°16’ 48”	+ 17”	83°17’ 05”
3	26° 53’40”	+ 16”	26° 53’56”

Check Sum = 180° 00’ 00”

Calculate the Azimuth of each side of the traverse

Key points

•Use the adjusted or balanced angles

•Requires knowledge of the traverse configuration or the direction of travel around the traverse. I.e., is the order of stations clockwise or counter clockwise as you proceed from station to station?

Calculate the Azimuth of each side of the traverse (cont.)

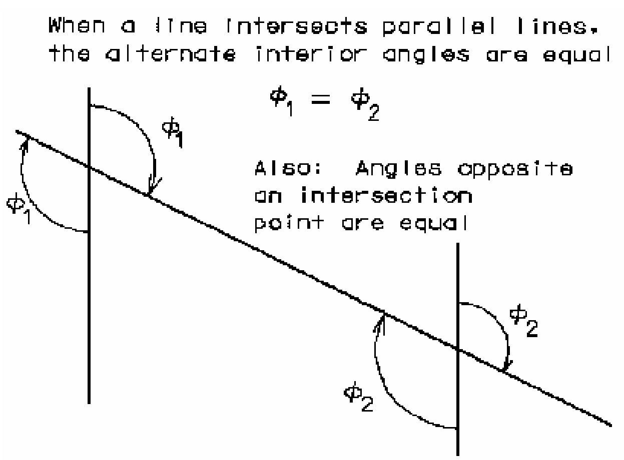
Key points cont.

•Requires knowledge of different horizontal angles - int., ext., deflection angle, angle-to-the-right, etc.

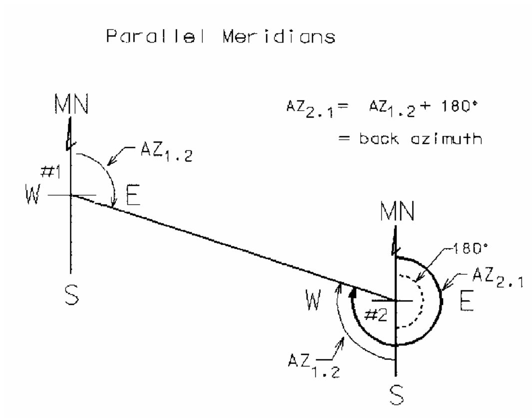
•Meridians are parallel

•Line intersecting parallel lines creates equal alternate interior angles (see Geometry Review)

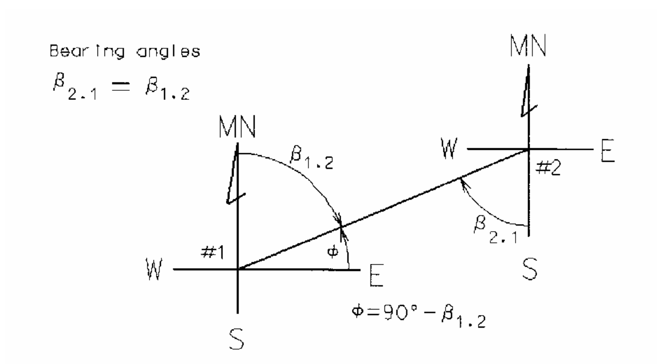
Geometry Review



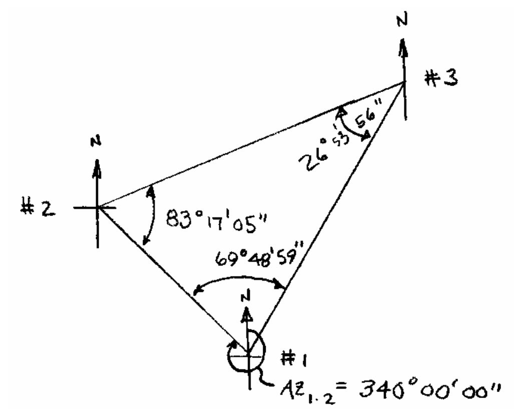
Review (cont.)



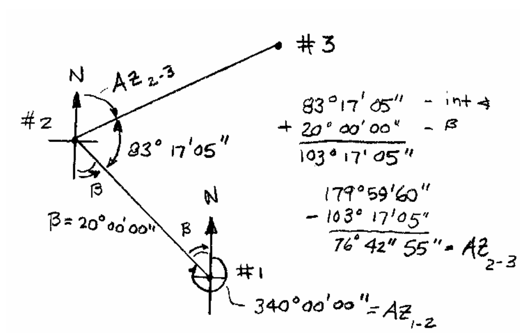
Review (cont.)



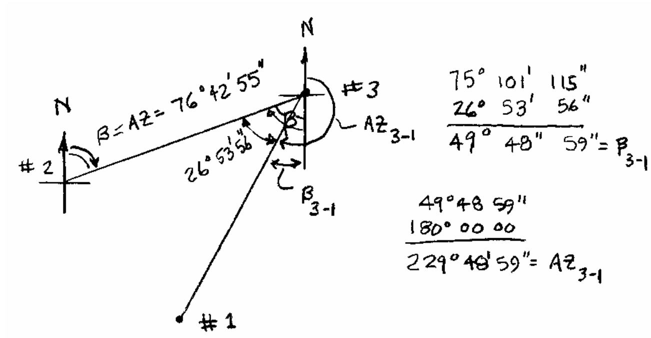
Calculate the Azimuth of each side - using Adjusted Int. Angles



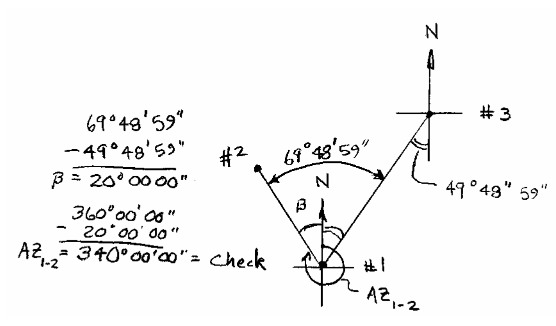
Find the Azimuth of side 2-3



Find the Azimuth of side 3-1

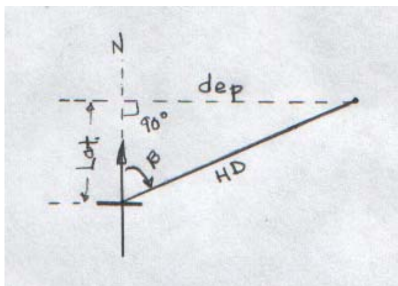


Calculate azimuth - check 1 -2

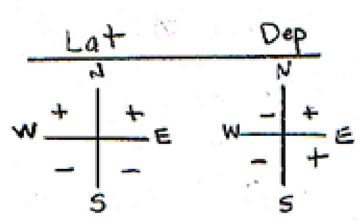


Calculate Latitude and Departure

- Latitude = lat = HD · Cos(β) = HD · Cos(Az)
- Departure = dep = HD · Sin (β) = HD· Sin (Az)



Sign Convention

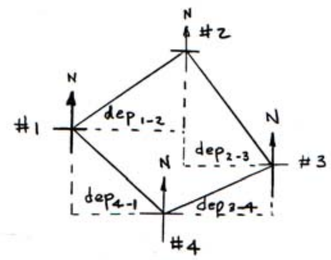


Latitudes and Departures (cont.)

Traverse type	**Mathematical Condition**	
	latitudes	departures
Loop	Σ lat = 0	Σ dep = 0
Connecting	Σ lat = ΔN	Σ dep = ΔE

Latitude and Departure (cont.)

Loop Traverse

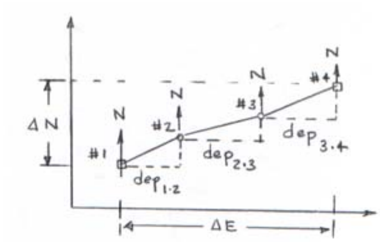


$\Sigma dep = dep_{1-2} + dep_{2\text{-}3} + dep_{3\text{-}4} + dep_{4\text{-}1} = 0$

$\Sigma Lat = 0$

Latitude and Departure (cont.)

Connecting Traverse



$\Sigma dep = dep_{1\cdot 2} + dep_{2\cdot 3} + dep_{3\cdot 4} = \Delta E$

$\Sigma lat = \Delta N$

Lat and Dep (cont.)

Example Calculation for lines 1-2 and 2-3

- Latitude = lat = HD · Cos(β) = HD · Cos(Az)
- Departure = dep = HD · Sin (β) = HD· Sin (Az)

$$\textbf{lat} = \textbf{104.919} \cdot \textbf{cos}(\textbf{340}^\circ \textbf{00'00"}) = +\textbf{98.592}$$

$$\textbf{dep} = \textbf{104.919} \cdot \textbf{sin}(\textbf{340}^\circ \textbf{00'00"0} = -\textbf{35.884}$$

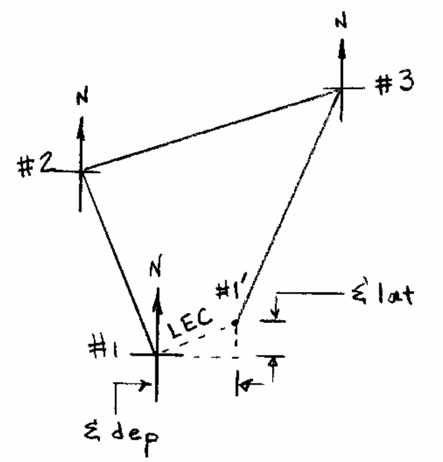
$$\textbf{lat} = \textbf{217.643} \cdot \textbf{cos}(\textbf{76}^\circ \textbf{42'55"}) = +\textbf{50.012}$$

$$\textbf{dep} = \textbf{217.643} \cdot \textbf{sin}(\textbf{76}^\circ \textbf{42'55"}) = +\textbf{211.819}$$

Lat and Dep (cont.)

Line	Az	HD (ft.)	Lat (ft.)	Dep (ft.)
1-2	340°00’00”	104.919	98.592	- 35.884
2-3	076°42’55”	217.643	50.012	211.819
3-1	229°48’59”	230.222	- 148.548	-175.885
	Totals	552.784	+ 0.056	+ 0.050

Calculate L.E.C. and R.E.C.



Loop Traverse

Closure lat = Σ **lat** − **0** = Σ **lat**

Closure dep = Σ **dep** − **0** = Σ **dep**

$$\mathbf{L.E.C} = \sqrt{(\Sigma\ \mathbf{lat})^2 + (\Sigma\ \mathbf{dep})^2}$$

$$\mathbf{R.E.C.} = \frac{\mathbf{L.E.C.}}{\sum \mathbf{HD_i}} = \frac{\mathbf{1}}{\mathbf{n}}$$

L.E.C. and R.E.C. (cont.)

• The R.E.C. should meet the standard of accuracy required for the traverse specification being followed. See ALTA-ACSM specifications for boundary traverses for an example.

• If the R.E.C. does not meet the standard of accuracy, a gross blunder and/or an unaccounted for systematic error among the measurements should be isolated and corrected before continuing with further adjustments.

L.E.C. and R.E.C. (cont.)

• If angle closure was satisfactory, a failure to meet the standard of accuracy for the R.E.C. at this point, likely points to a problem with distances.

L.E.C. and R.E.C. (cont.)

Loop Traverse

$$\textbf{Closure lat} = \Sigma\ \textbf{lat} - \textbf{0} = \Sigma\ \textbf{lat}$$

$$\textbf{Closure dep} = \Sigma\,\textbf{dep} - \textbf{0} = \Sigma\,\textbf{dep}$$

$$\textbf{L.E.C} = \sqrt{(\Sigma\ \textbf{lat})^{2} + (\Sigma\ \textbf{dep})^{2}}$$

$$\textbf{R.E.C.} = \frac{\textbf{L.E.C.}}{\sum \textbf{HD}_{\textbf{i}}} = \frac{\textbf{1}}{\textbf{n}}$$

L.E.C. and R.E.C. (cont.)

Connecting Traverse

$$\textbf{Closure lat} = \Sigma\ \textbf{lat} - \Delta\textbf{N} = \textbf{E}_\textbf{L}$$

$$\textbf{Closure dep} = \Sigma\,\textbf{dep} - \Delta\,\textbf{E} = \textbf{E}_\textbf{D}$$

$$\textbf{L.E.C} = \sqrt{(\textbf{E}_\textbf{L})^2 + (\textbf{E}_\textbf{D})^2}$$

$$\textbf{R.E.C.} = \frac{\textbf{L.E.C.}}{\sum \textbf{HD}_\textbf{i}} = \frac{\textbf{1}}{\textbf{n}}$$

L.E.C. and R.E.C. (cont.)

Example Calculation

Loop Traverse

$$\textbf{Closure lat} = \Sigma\ \textbf{lat} - \textbf{0} = \Sigma\ \textbf{lat} = +0.056$$

$$\textbf{Closure dep} = \Sigma\,\textbf{dep} - \textbf{0} = \Sigma\,\textbf{dep} = +0.050$$

$$\textbf{L.E.C} = \sqrt{(\Sigma\ \textbf{lat})^2 + (\Sigma\ \textbf{dep})^2} = \sqrt{(0.056)^2 + (0.050)^2} = 0.075$$

$$\textbf{R.E.C.} = \frac{\textbf{L.E.C.}}{\sum \textbf{HD}_\textbf{i}} = \frac{0.075}{552.784} = \frac{1}{7370}$$

Traverse Adjustment

- Adjustment Methods to remove random errors
  – Compass Rule*
  – Transit Rule
  – Crandall Method
  – General Least Squares Method

Traverse Adjustment via Compass Rule

$$C_{Li} = -\frac{\sum \text{lat}}{\sum \text{HD}} \cdot \text{HD}_i = \text{correction to a latitude}$$

$$\text{note: } -\frac{\sum \text{lat}}{\sum \text{HD}} = \text{Correction per ft.}$$

$$C_{Di} = -\frac{\sum \text{dep}}{\sum \text{HD}} \cdot \text{HD}_i = \text{correction to a departure}$$

$$\text{note: } -\frac{\sum \text{dep}}{\sum \text{HD}} = \text{Correction per ft.}$$

$$\text{bal. lat.} = \text{lat.} + C_L$$

$$\text{bal. dep.} = \text{dep.} + C_D$$

Traverse Adjustment (cont.)

Example calculation for line 1-2

$$\mathbf{C_L = -\frac{0.056}{552.784} \cdot 104.919 = -0.011}$$

$$\mathbf{C_D = -\frac{0.050}{552.784} \cdot 104.919 = -0.009}$$

$$\mathbf{bal.\ lat. = 98.592 + (-0.011) = 98.581}$$

$$\mathbf{bal.\ dep. = -35.884 + (-0.009) = -35.893}$$

Traverse Adjustment (cont.)

Line	Lat. (ft.)	Dep. (ft.)	Correction (ft.)		Bal. Lat.(ft.)	Bal. Dep(ft.)
			Lat	Dep		
1-2	+98.592	-35.884	-0.011	-0.009	+98.581	-35.893
2-3	+50.012	+211.819	-0.022	-0.020	+49.990	+211.799
3-1	-148.548	-175.885	-0.023	-0.021	-148.571	-175.906
sums	+0.056	+0.050	-0.056	-0.050	0.000	0.000

Calculate Coordinates

Usually, the coordinates of the first point are assigned arbitrary values so other coordinates will be positive.

$$\mathbf{N_i = N_{i-1} + bal.\ lat_{i-1,\ i}}$$
$$\mathbf{E_i = E_{i-1} + bal.\ dep_{i-1,\ i}}$$

$$\mathbf{N_2 = N_1 + bal.\ lat_{1-2} = 1000.000 + 98.581 = 1098.581}$$
$$\mathbf{E_2 = E_1 + bal.\ dep_{1-2} = 1000.000 + (-35.893) = 964.107}$$

Calculate Coordinates (cont.)

$$\mathbf{N_i = N_{i-1} + bal.\ lat_{i-1,\ i}}$$
$$\mathbf{E_i = E_{i-1} + bal.\ dep_{i-1,\ i}}$$

Sta	Bal. Lat. (ft.)	Bal. Dep. (ft.)	Coordinates (ft.)	
			Northing	Easting
1			1000.000	1000.000
	+98.581	-35.893		
2			1098.581	964.107
	+49.990	+211.799		
3			1148.571	1175.906
	-148.571	-175.906		
1			1000.000	1000.000

Calculate Area by Coordinates

$2\,\text{Area} = E_1 \cdot (N_n - N_2) + E_2 \cdot (N_1 - N_3) + \ldots E_n \cdot (N_{n-1} - N_1)$

- where n = number of sides
- a term is written for each vertex or traverse station
- parenthetical term = (preceding N – following N)

Area by Coordinates (cont.)

For computational convenience the terms are written in a vertical stack. An example for a 3-sided traverse is as follows:

$E_1 \cdot (N_3 - N_2)$
$E_2 \cdot (N_1 - N_3)$
$E_3 \cdot (N_2 - N_1)$

$\Sigma = 2\,\text{Area}$

Area by Coordinates (cont.)

Example calculation:

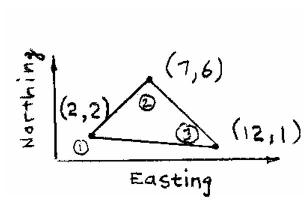
$$\mathbf{1000.000 \cdot (1148.571 - 1098.581) = 1000.000 \cdot (49.990) = 49990.0000}$$
$$\mathbf{964.107 \cdot (1000.000 - 1148.571) = 964.107 \cdot (-148.571) = -143238.3411}$$
$$\mathbf{1175.906 \cdot (1098.581 - 1000.00) = 1175.906 \cdot (98.581) = \underline{115921.9894}}$$
$$\mathbf{2\ Area = 22673.6483\ ft.^2}$$

Area = 22,673.6483 ft^2 ÷ 2 = 11,336.8242 ft^2

Area in acres = 11,336.8242 ft^2 ÷ 43560 ft^2/acre = 0.26 acres

Alternative Coordinate Method

Determinate Method



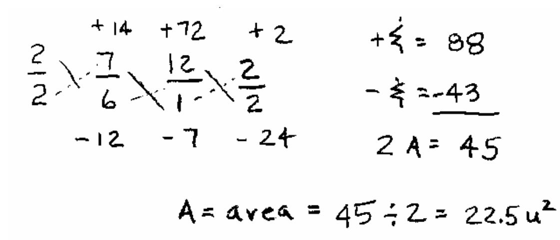
Setup coordinate pairs to look like fractions - eastings over northings, and in sequence around the traverse. NOTE: the first coordinate pair is repeated at the end.

$$\frac{2}{2} \quad \frac{7}{6} \quad \frac{12}{1} \quad \frac{2}{2}$$

Determinate Method (cont.)

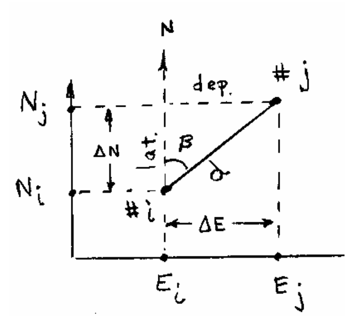
• Obtain cross products, “up” products are multiplied by a plus one and “down” products are multiplied by a negative one.

• The algebraic sum of the cross products is twice the area.

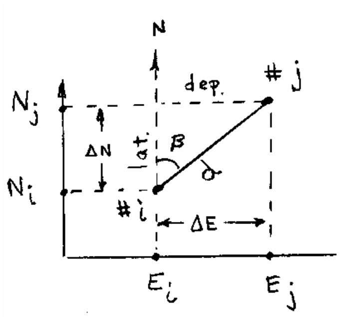


Inverse Problem

Inverse Problem - Given the coordinates of two points (i and j) find: the dir. and dist. between them.



Inverse Problem



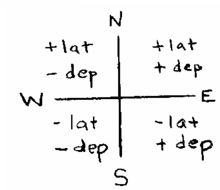
$$\mathbf{lat}_{i.j} = \Delta\mathbf{N}_{i.j} = \mathbf{N}_j - \mathbf{N}_i$$

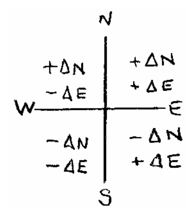
$$\mathbf{dep}_{i.j} = \Delta\mathbf{E}_{i.j} = \mathbf{E}_j - \mathbf{E}_i$$

$$\beta_{i.j} = \mathbf{arctan}\left|\frac{\mathbf{dep}_{i.j}}{\mathbf{lat}_{i.j}}\right| = \mathbf{arctan}\left|\frac{\Delta\mathbf{E}_{i.j}}{\Delta\mathbf{N}_{i.j}}\right|$$

$$\mathbf{d}_{i.j} = \sqrt{\mathbf{dep}_{i.j}^2 + \mathbf{lat}_{i.j}^2} = \sqrt{\Delta\mathbf{E}_{i.j}^2 + \Delta\mathbf{N}_{i.j}^2}$$

Determine Quadrant by Inspection





Inverse Problem (cont.)

Sample Calculation for line 1-2

$$\mathbf{lat}_{1.2} = \Delta\mathbf{N}_{1.2} = \mathbf{1098.581 - 1000.000} = +\mathbf{98.581}$$

$$\mathbf{dep}_{1.2} = \Delta\mathbf{E}_{1.2} = \mathbf{964.107 - 1000.000} = -\mathbf{35.893}$$

$$\beta_{1.2} = \mathbf{arctan}\left|\frac{-\ \mathbf{35.893}}{+\ \mathbf{98.581}}\right| = \mathbf{20^\circ 00'23''}$$

$$\mathbf{d}_{i.j} = \sqrt{(-\mathbf{35.893})^2 + (+\mathbf{98.581})^2} = \mathbf{104.912}$$

Note: The bearing quadrant is NW; therefore, the bearing is: N 20°00’23”W or the Azimuth is 339°59’37”

Inverse Problem (cont.)

Find the corrected Azimuth and Distance between traverse stations by inversing.

Example problem:

Line	Corrected Az	Corrected dist. (ft.)
1-2	339°59’37”	104.912
2-3	076°43’11”	217.619
3-1	229°48’55”	230.253