Mat. Koe kurssi 10 23.9.-08

1. a) $\frac{3x^2}{2} - x + C$

b) $\int (2x-1)^{\frac{1}{3}} dx$

$= \frac{1}{2}\int 2(2x-1)^{\frac{1}{3}} dx$

$= \frac{1}{2} \frac{(2x-1)^{\frac{4}{3}}}{\frac{4}{3}} + C$

$= \frac{3}{8}(2x-1)\sqrt[3]{2x-1} + C$

c) $\int_{-1}^{2} e^{2x} dx = \frac{1}{2} \Big/_{-1}^{2} e^{2x}$

$= \frac{1}{2}(e^4 - e^{-2})$

2. $F(x) = -\frac{6x^3}{3} + \frac{4x^2}{2} + C$

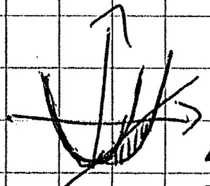
$= -2x^3 + 2x^2 + C$

$F(1) = -2$

$F(1) = -2 + 2 + C = -2$

$C = -2$

$F(x) = -2x^3 + 2x^2 - 2$

3a) 

$3x-2 = x^2-2$

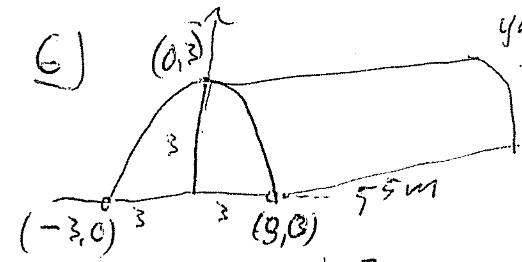
$x^2 - 3x = 0$

$x = 0$ tai $x = 3$

$A = \int_0^3 (3x-2) - (x^2-2)\, dx$

$= \int_0^3 -x^2 + 3x\, dx = 4{,}5$

3b alhaalla

6) 

yhtälö:

$f(x) = a(x-3)(x+3)$

$= a(x^2-9)$

$f(0) = a(0-9) = 3$

$a = -\frac{1}{3}$

siis $f(x) = -\frac{1}{3}x^2 + 3$

$A = 2\int_0^3 -\frac{1}{3}x^2 + 3\, dx = 2 \Big/_0^3 -\frac{x^3}{9} + 3x = 2(-3+9-0)$

(parillinen) $= 12$

$V = Ah = 12 \cdot 55 = 660\ (m^3)$

4. $\int_a^{a+1} (2x+3)\, dx = \frac{1}{2}$

$\Big/_a^{a+1} (x^2 + 3x)$

$= (a+1)^2 + 3(a+1) - a^2 - 3a$

$= a^2 + 2a + 1 + 3a + 3 - a^2 - 3a$

$2a + 4 = \frac{1}{2}$

$2a = -\frac{7}{2}$

$a = -\frac{7}{4}$

$a = -1\frac{3}{4}$

salhaalla

7. $f(x) = \begin{cases} x-1, & x \geq 1 \\ -x+1, & x < 1 \end{cases}$

$-2 + \frac{1}{2} = -\frac{3}{2}$

on jatkuva, joten on olemassa integraalifunktio

$F(x) = \begin{cases} \frac{x^2}{2} - x + C, & x \geq 1 \\ -\frac{x^2}{2} + x + D, & x < 1 \end{cases}$

Koska F(x) on derivoituva, niin se on myös jatkuva.

$F(1) = \frac{1}{2} - 1 + C = -\frac{1}{2} + C$

$\lim_{x \to 1^-} F(x) = -\frac{1}{2} + 1 + D = D + \frac{1}{2}$

$D + \frac{1}{2} = C - \frac{1}{2}$

$D = C - 1$

$F(-1) = 0$

$-\frac{1}{2} - 1 + D = 0$

$D = \frac{3}{2}$

$C - 1 = \frac{3}{2}$

$C = \frac{5}{2}$

V: $F(x) = \begin{cases} \frac{x^2}{2} - x + 2{,}5, & x \geq 1 \\ -\frac{x^2}{2} + x + 1{,}5, & x < 1 \end{cases}$

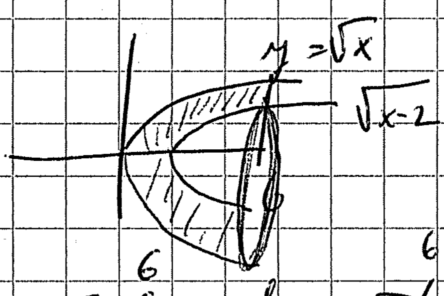
8. $y' = 2x + 1$

$y = x^2 + x + C$

$(-3, 0)$: $9 - 3 + C = 0$

$C = -6$

V: $f(x) = x^2 + x - 6$

9. 

$V_1 = \pi \int_0^6 x\, dx = \pi \Big/_0^6 \frac{x^2}{2} = \pi 18$

$V_2 = \pi \int_2^6 (x-2)\, dx = \pi \Big/_2^6 \left(\frac{x^2}{2} - 2x\right) = 8\pi$

$V = V_1 - V_2 = 10\pi$

3b) $\int_{\frac{1}{2}}^{1} \left(x - \frac{1}{x}\right)^2 dx = \int_{\frac{1}{2}}^{1} \left(x^2 - 2 + \frac{1}{x^2}\right) dx$

$= \Big/_{\frac{1}{2}}^{1} \left(\frac{x^3}{3} - 2x - \frac{1}{x}\right) = \left(\frac{1}{3} - 2 - 1\right) - \left(\frac{1}{24} - 1 - 2\right)$

$= \left(-3 + \frac{1}{3}\right) - \left(-3 + \frac{1}{24}\right)$

$= -\frac{8}{3} + \frac{71}{24} = -\frac{64}{24} + \frac{71}{24} = \frac{7}{24}$

5. $\int_a^{a+\pi} \cos^2 x\, dx = \int_a^{a+\pi} \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$

$= \Big/_a^{a+\pi} \left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right) = \frac{1}{2}(a+\pi) + \frac{1}{4}\sin 2(a+\pi)$

$- \left(\frac{1}{2}a + \frac{1}{4}\sin 2a\right)$

$= \frac{1}{2}\pi$ ei riipu a:sta

Koska $\sin 2a = \sin(2a + 2\pi)$