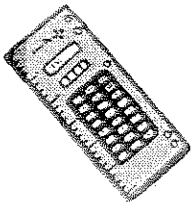
Real Life Math



Two runners are going to compete in a race and the officials have called upon you as their resident mathematician to help. Since the runners must stay in their respective lanes, the officials need you to help them determine the handicap that should be given the runner in lane 2. Lane widths are 4 feet.

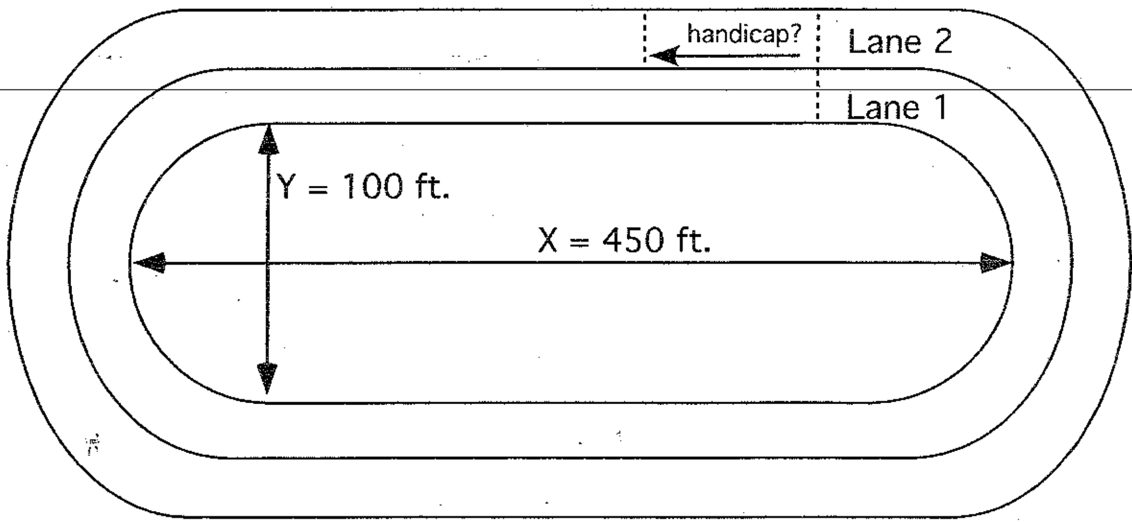
a) *What handicap would you suggest for the runner in the outer lane?*

<u>Make a conjecture about the next three questions before you answer them</u>

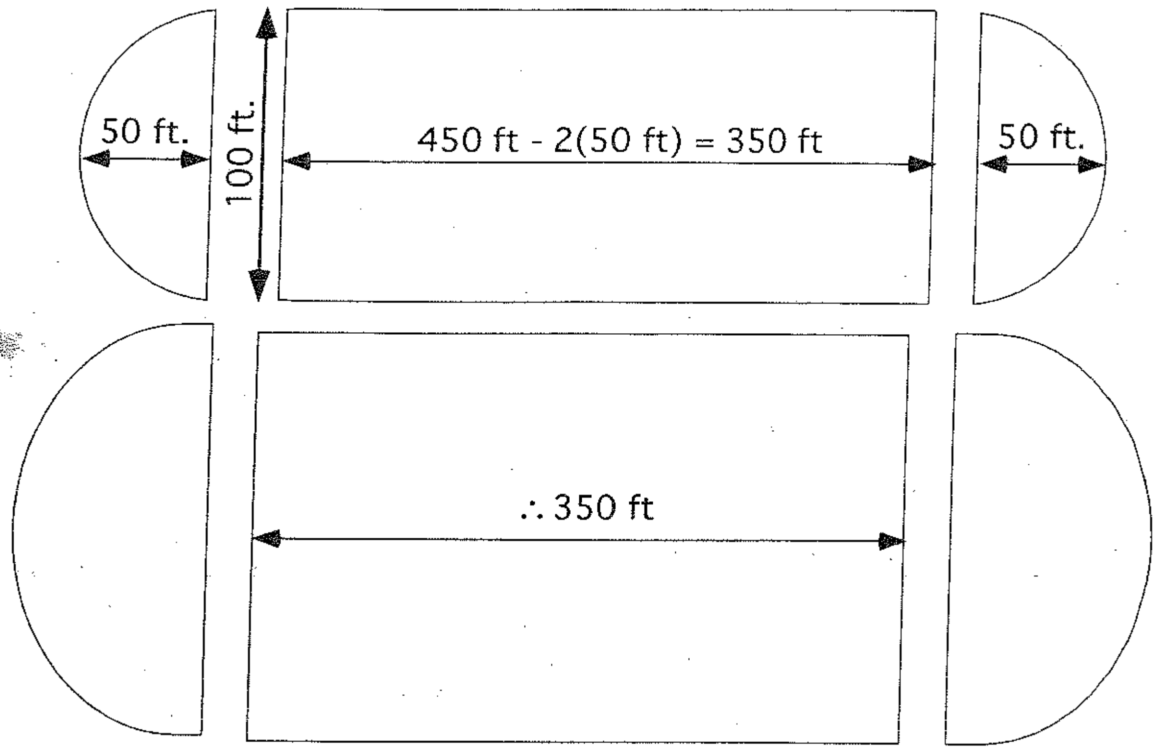
b) Does the diameter of the semi-circle (Y) influence the handicap?

c) Does the lenqth of the track (X) influence the handicap?

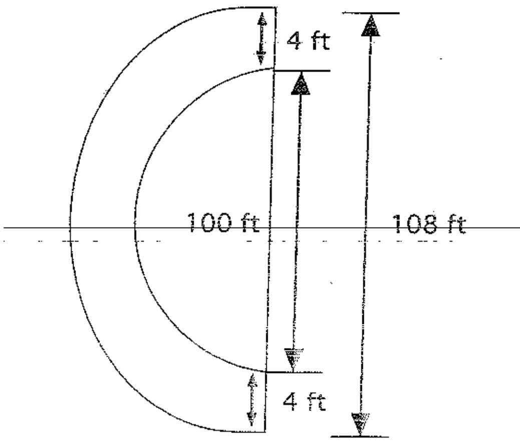
Justify your answers.



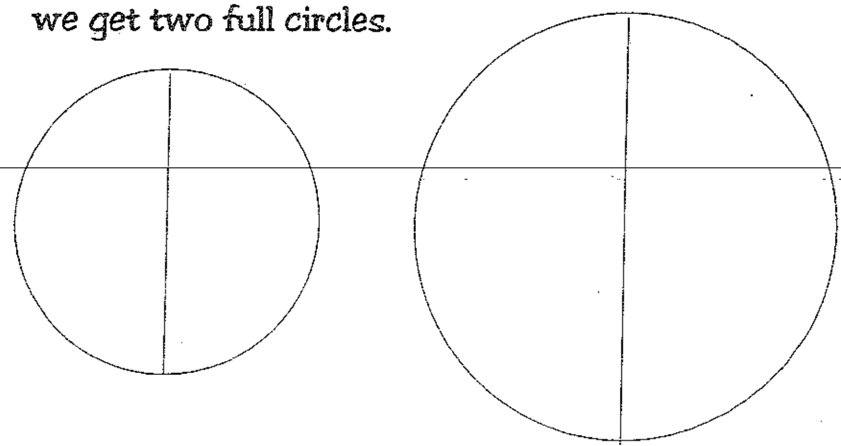
a) X = the full length of the track = 450 ft. Y = the width of the track = 100 ft.
Since the diameter of the small semi-circle is 100 ft., the radius is 50 ft.
We have two rectangles, both have the same length 450 ft. - 2(50 ft.) = 350 ft.
They will run the same distance laterally in the image.



The different distances they will run will be along the sides of the track ... the distances around the semi-circles.
The diameter of the outer semi-circle is 8 ft. greater than the diameter of the inner semi-circle, or a total of 108 ft.



If we combine the two end semi-circles from each lane, we get two full circles.



The runner on the outside track has to run a greater distance than the runner on the inside track. We can calculate that distance by determining the circumferences of the two circles and subtracting the smaller from the larger.
Using the formula $C = \pi D$, the circumference of the larger circle is $C = 108\pi = 339.12$ ft.
The circumference of the smaller circle is $C = 100\pi = 314$ ft.
The difference between them is approximately 25.12 feet.
The runner in the outside circle should have a headstart of 25.12 feet.

b) Several approaches can be used to demonstrate changing the width of the track does not affect the difference between the two paths.

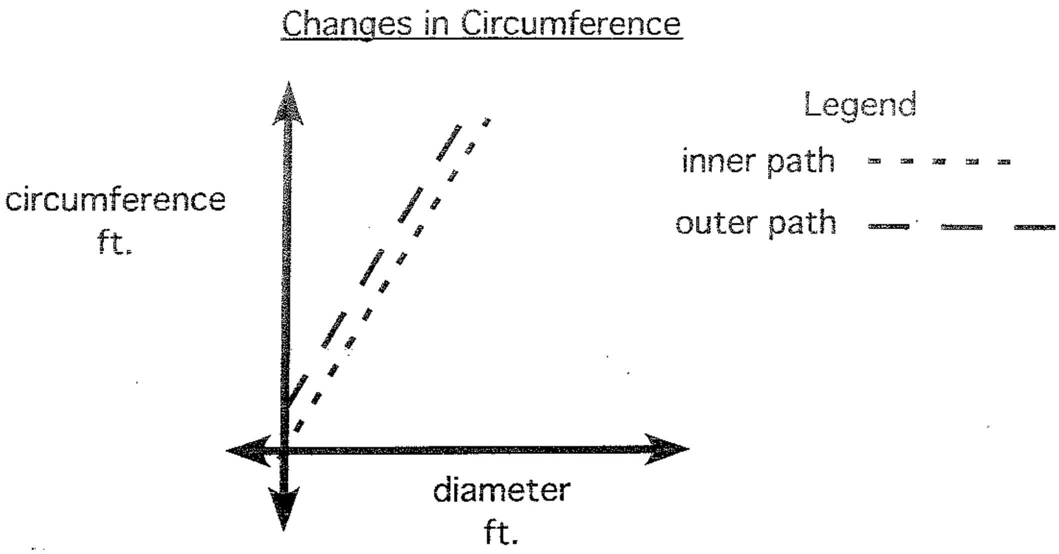
One approach is to use the circumference formula: $C = \pi D$. The circumference of the joined smaller semi-circles is simply $C = \pi D$, where D is the diameter of the smaller circle. The diameter of the combined larger semi-circles will be 8 ft. larger than the smaller one. The circumference of the joined larger semi-circles will be $C = \pi(D + 8)$, or $C = \pi D + 8\pi$. The difference in circumferences will be the constant, 8π, or approximately 25.12 feet.

A second approach is to create a table using a variety of different diameters with a constant difference between the two diameters of 8 ft. The first column lists a variety of diameters for the inner path while the second column lists the corresponding diameters for the outer path. The third and fourth columns show the respective circumferences. The last column shows the difference between the two paths. The table demonstrates there is a constant difference between the two paths. Hence, changing the width of the track will not influence the length of the headstart needed by the runner on the outer path.

Diameter Y (inner path)	Diameter of outer path	Circumference of inner path	Circumference of outer path	Difference between the two paths
20	28	62.8	87.92	25.12
30	38	94.2	119.32	25.12
40	48	125.6	150.72	25.12
50	58	157	182.12	25.12
60	68	188.4	213.52	25.12
70	78	219.8	244.92	25.12
80	88	251.2	276.32	25.12
90	98	282.6	307.72	25.12
100	108	314	339.12	25.12
110	118	345.4	370.52	25.12
120	128	376.8	401.92	25.12
130	138	408.2	433.32	25.12
140	148	439.6	464.72	25.12
150	158	471	496.12	25.12
160	168	502.4	527.52	25.12
170	178	533.8	558.92	25.12
180	188	565.2	590.32	25.12
190	198	596.6	621.72	25.12
200	208	628	653.12	25.12

Another approach that demonstrates distance is constant can be done by graphing the two equations for the circumferences of each circle.

The equation for the circumference of the joined semi-circles for the inner path is $C_{inner} = \pi D$.
The equation for the circumference of the joined semi-circles for the outer path is $C_{outer} = \pi(D + 8)$.



The result is two parallel lines. Hence, the distance remains constant.

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c) Changing the value the length of the track will not influence the handicap since the lengths of both rectangles always remain the same.