Complex Dynamical Systems and the Social Sciences

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December 13, 2007

Abstract

An agent-based modeling environment, NetLogo, is the subject of this chapter. We review recent activity in social science research in which NetLogo has had a role, including three projects of our own.

1 Introduction to NetLogo

The symbiosis of mathematics and the sciences has been described as an hermeneutical circle. That is, experimental data determines a model, the model suggests new experiments, new data refines the model, and so on. This infinite loop is the motor for the advance of science.

Mathematics and the sciences became joined in this hermeneutical circle four hundred years ago, when mathematical modeling came of age: mathematical physics since 1600, mathematical biology since 1930 or so, and the mathematical social sciences, primarily, since the 1950s. This latter has exploded since the advent of agent-based modeling (ABM), a tool for the modeling and simulation of complex dynamical systems. ABM may be regarded as an evolution of object-oriented programming (OOP). Whereas OOP introduced objects, reusable modules, protected data, interfaces, and so on, in ABM we have objects which may act as independent agents. Chris Langton, of Artificial Life fame, pioneered the way with his Swarm language at the Santa Fe Institute in the 1980s. An early application considered the native cultures of the American Southwest as a complex dynamical system.

A number of programming environments for agent-based modeling have been developed recently, mostly by graduate schools of the social sciences, and there are now many. We may mention six:

- Ascape, Center on Social and Economic Dynamics, The Brookings Institution
- Mason, Center for Social Complexity, George Mason University
- NetLogo, Center for Connected Learning, Northwestern University
- Repast, Social Science Research Computing, University of Chicago
- StarLogo TNG, Media Laboratory, Massachusetts Institute of Technology
- Swarm, Center for the Study of Complex Systems, University of Michigan

In this chapter we will focus on the one that we have used extensively, NetLogo.

NetLogo evolved from Logo, the language developed 1n 1967 by Wally Feurzeig and Seymour Papert, cofounder of the AI Lab and the Media Lab at MIT, to teach programing concepts to children. After the ABM concept emerged, Logo evolved into StarLogo, and then into NetLogo, by Uri Wilensky in 1999. Quoting from http://ccl.northwestern.edu/netlogo/faq.html:

The original StarLogo was developed at the MIT Media Lab in 1989-1990 and ran on a massively parallel supercomputer called the Connection Machine. A few years later (1994), a simulated parallel version was developed for the Macintosh computer. That version eventually became MacStarLogo. StarLogoT (1997), developed at the Center for Connected Learning and Computer-Based Modeling (CCL), is essentially an extended version of MacStarLogo with many additional features and capabilities. Since then two multi-platform Java-based multi-agent Logos have been developed: NetLogo (from the CCL) and a Java-based version of StarLogo (from MIT).

2 Survey of recent literature

The social sciences have been pumped into an excited state by the sudden arrival of the mathematical modeling tool of dreams, ABM. Consulting the model libraries at the NetLogo website, we find sample models in the categories: Art, Biology, Chemistry & Physics, Computer Science, Earth Science, Games, Mathematics, Networks, Social Science, and System Dynamics. And under Social Science we find the exemplary models: AIDS, Altruism, Cooperation, Ethnocentrism, Party, Rumor Mill, Scatter, Segregation, Simple Birth Rates, Traffic, Voting, Wealth Distribution, Cash Flow, Prisoner's Dilemma, and several more. Finally, hundreds of additional models may be found in the library of NetLogo User Community Models maintained on the NetLogo website.

The hermeneutical hope of these mathematical modeling projects is to discover rules for the behavior of social systems. In physics, we have the archetype of Galileo rolling billiard balls down inclined tracks, collecting data points timed with his isochronous pendulum, and fitting a quadratic curve. In the social sciences we have no Newton's law, $F = ma$, but if we can fit behavioral data with a model, we may discover a rule. ABM makes this easy, and yet for a successful computer model there may be no corresponding mathematics.

3 Landscape dynamics

Advances in mathematics in the 1960s made available a host of new modeling strategies for all the sciences. The framework of global analysis applied to dynamical systems, calculus of variations, partial differential equations of evolution type, game theory, and so on, brought us catastrophe theory, chaos theory, complexity and simplexity, neural network theory, evolutionary game theory, and others. The computer and computer graphic revolutions brought new possibilities of computational modeling, simulation, and scientific visualization. Of all the sciences, those with the greatest potential to benefit from these new methods are the social, behavioral, and economic sciences.

One effort to embed a social model into a mathematical framework is the gradient hill-climbing scheme called landscape dynamics by one of the authors in the 1990s. This is a mathematical modeling and computer simulation technology based on evolutionary games with continuous spaces of strategies.

It is an application of global analysis methods to game theory. It extends the class of models called evolutionary games and opens it to new applications in the social sciences. Landscape dynamics is intended to advance the arts of mathematical modeling, computer simulation, and scientific visualization of complex dynamical systems encountered in the social, behavioral, and economic sciences.

Evolutionary game models analyze strategic interaction over time. Equilibrium emerges, or fails to emerge, as players adjust their strategies in response to the payoffs they earn. Early models have mainly considered situations in which players chose among only a few discrete strategies. Landscape dynamics allows players to choose within a continuous strategy space, A.

In this setup, the current state is the distribution of all players choices over A. In any particular application, the current state defines a payoff function on A, whose graph is called the adaptive landscape. Players respond to the landscape in continuous time by adjusting their strategies towards higher payoff. Hence the current state (the distribution of chosen strategies) changes, and this in turn alters the landscape. The interplay between the evolving state and the landscape gives rise to nontrivial dynamics. In particular, when players follow the gradient (steepest ascent in the adaptive landscape), the evolving state can be characterized as the solution to a nonlinear partial differential equation, or equivalently, a dynamical system on an infinite-dimensional space.

For more details, consult the articles on the Landscape Dynamics website (below). This material is based upon work supported by the National Science Foundation under Grant No. 0436509.

4 Conspicuous consumption

In 2005, landscape dynamics became a research project of the authors at the University of California at Santa Cruz, with funding from the National Science Foundation. Our first application in this project is a model for Veblen's notion of conspicuous consumption. This is an exemplary model that shows our approach in the context of an intuitive and well-known problem. There are two basic flavors of conspicuous consumption, as described in the literature: envy and pride. For the present, we restrict attention to the envy model. In our implentation of the model, the initial distribution may be constructed.

4.1 The initial setting

The strategy space, or action set, A, is the unit interval, $[0, 1]$, in this model. The agents, called turtles in NetLogo, represent consumers. Each consumer is shown as a triangle on the strategy space. (The full graphic user interface of this model is shown in Figure 1.) They have different colors just for the visual effect. When several consumers are on the same patch (discretized interval of the strategy space) only the top one can be seen in entirety, but the horizontal position is a floating point number, so parts of lower turtles may be seen.

The strategy space is shown as five horizontal rows in the upper half of the graphics window (the black rectangle in the interface. These are to be regarded as superimposed layers on a single interval. It represents a unit interval corresponding to the choice of strategy, x. All consumers have the same income, 1, but choose variously how much to spend on ordinary consumption, x, and how much to spend on conspicuous consumption, $1 - x$. Thus $x = 0$ represents 100% conspicuous consumption, such as diamond rings, and $x = 1$ represents 100% discrete consumption, such as savings.

A chosen number of consumers begin at initial positions in the strategy space. This initial density is important to the outcome of a run. This model is arranged so that the initial density is the sum of an arbitrary number of square waves. Thus the operator may approximate an arbitary initial density. Interesting choices include a single square wave or herd, two herds, a tent shape or heap, two heaps, and so on. In any case, the operator begins by adding square waves, or sub-herds, until a desired initial distribution is obtained. Each addition of a sub-herd is called a "puff".

4.2 The distribution, $F(x)$

The instantaneous state of the system is represented by the density of consumers in the strategy space, f), a probability measure, or equivalently by its cumulative distribution, F, the integral of f, a monotone function increasing from zero to one.

The density, f, is also shown as a graph in the upper plot window in the interface, labelled "Density of Consumers", showing the average density of turtles on each patch.

4.3 The payoff, $\phi(x, F)$

The most important function in the model is the payoff function, or fitness function, ϕ. It is a real-valued function, depending on both x and F. The function ϕ is the landscape in this example of landscape dynamics. The definition of ϕ used here, called the *envy rule*, is the sum of two nonpositive functions:

$$c \log(x)$$

negative as x is in the interval $(0, 1]$, and

$$-\int_0^x F(y)dy$$

negative as $F(y)$ is nonnegative. Note that we avoid the troublesome value $x = 0$. The graph of ϕ is shown as below the graph of f, and on the same horizontal scale, $(0, 1]$. The two plots are updated after every 10th step.

4.4 The slope

The slope of the landscape, or gradient of ϕ, ϕ_x, also called the fitness gradient, is given by the formula:

$$\phi_x(x, F) = c/x - F(x).$$

The first term in the envy rule is the direct utility a consumer receives from ordinary consumption. It is monotone decreasing to the value c at $x = 1$. The constant c may be set with a slider. It represents the importance of ordinary consumption relative to conspicuous consumption. The second term is also monotone decreasing. We will be especially interested in the zero-crossing of this function. Here the slope is shown with colors on the color bar below the gray row in the black screen. Its color code is:

- $+0.1$ or higher [red], positive (meaning step to the right)
- -0.1 to 0.1 [yellow], close to zero (small step to the left or right)
- -0.1 or lower [green], negative (step to the left)

This is chosen to emphasize the zero-crossing of the slope. As the slope depends on F, which is time-dependent, the yellow segment of the slope color bar will be expected to move about.

4.5 The step

Consumers step uphill on the landscape. With each increment of discrete time, each consumer adjusts her strategy, x, by an increment proportional to the slope. The proportion (stepsize) may be set with a slider. Thus each turtle moves uphill by an increment: stepsize * slope. This is the Euler method for integrating the partial differential equation representing the envy rule.

4.6 The behavior of the model

Simulations with our NetLogo model conform to the expected result: all the consumers converge to a single attractive strategy. This is expected because of a convergence theorem, which has been obtained using global analysis. (Friedman and Ostrov, 2008) This theorem is possible because landscape dynamics is a bridge between agent-based modeling and pure mathematics. The agreement between the behavior of the simulations and the conclusion of the theorem provides a degree of validation of our NetLogo model.

5 Financial markets

A series of NetLogo models for financial markets has been developed under our three-year grant. Further background, and the models themselves, may be found at the

Landscape Dynamics website (below). The goal has been to discover the dynamics underlying the phenomena of bubbles and crashes. Here we will describe the simplest of our models. This is joint work with our students, Matt Draper, Don Carlisle, and Ken Van Haren.

5.1 The setting

The state space is shown in a graphics window. The horizontal axis represents a unit interval corresponding to the choice of strategy, u. This is the degree to which the manager is willing to invest in risky assets. Moving to the right increases risk. The vertical axis represents the value of the manager's portfolio, z. The upper limit is set by a slider with default value 4. A portfolio value of one is considered normal. (The interface is shown in Figure 2.)

The agents (turtles) represent money market managers. Each manager is shown as a small triangle in the graphics window. A chosen number of managers begin at initial positions in the state space. The initial distribution is important to the outcome of a run. The model starts up with a random distribution in a rectangle of width, height, and position set by sliders.

5.2 The step

Stepsize is a unit of time for periodic reports of financial data. The stepsize may be set with a drop-down menu. For example, if "52" is chosen, this signifies a frequency of 52 (weekly) steps per year, and the variable "stepsize" in the program is set to 1/52 years. An additional parameter, "u-steps", may be set with a drop-down menu. This is the number of substeps in a step. Increasing u-steps decreases the substepsize, called "stepsize-u", to the ratio stepsize/u-steps, and decreases the numerical error in the Euler integration.

5.3 The payoff function

The payoff function is,

$$\phi(x, F) = x(R_1 - R_o) - \frac{1}{2}c_2 x^2.$$

where R_0 is the rate of return of the risk-free asset, R_1 that of the risky asset, and c_2 is a positive constant set by a slider.

Using the Euler algorithm, each manager is assumed to move horizontally up the slope of the payoff function by the substep increment, jump-u = stepsize-u * slope.

With every step (or u-step substeps) there is also a vertical motion due to increment or decrement of the size of the manager's portfolio due to payoffs. See Friedman (2007), and the documentation online at the landscape Dynamics website, under "Models".

6 Two-party voting

Harold Hotelling (1929) developed a seminal microeconomic theory of spatial competition in which firms tend to be attracted to the center of a one-dimensional bounded space. The classical Hotelling model illustrates, using a colorful example, that two competing hotdog firms on a one-dimensional stretch of beach (bounded at each end) tend to move toward the middle of the beach, given a uniform distribution of customers. Drawing from Hoteling's work, Black (1948) and Downs (1957) constructed spatial voting models. Black's model treats candidates as policy alternatives or points in Euclidean space; voters are the principal actors. In the Downsian tradition, under which the bulk of research in the spatial voting literature has been conducted, candidates are analogous to the firms in Hotelling's model. The result of the Downsian model is that if a voter supports the nearest candidate, then candidates seeking to win an election ultimately will locate themselves at the position of the median voter. In this regard, in Downsian models political competition in a two-party system is often viewed as a fight for the middle. A political agent, e.g., a party leader running for office, hopes to attract more votes by touting a center-of-the-road platform, as a large majority of potential votes lies near the "median voter." Since the early work of Black and Downs, the literature on spatial voting models has become quite rich with many variants and refinements.

The basic axioms of landscape dynamics are that myopic agents adopt a strategy in a continuous strategy space; such agents alter their strategies only incrementally and, by changing their own strategies, affect the payoffs and thus the strategies of their counterparts. Given these assumptions, we may set the stage for a particular application of landscape dynamics to a two-party voting model in which parties locate in a two-dimensional issue space and adjust their platforms incrementally so as to increase their vote shares. Suppose that two salient issues dominate the political discourse of the country: Issue X and Issue Y. We then array these issues on two distinct axes. Think of this Cartesian plane as an "issue space" in which political agents compete for blocks of voters. Each voter has an "attraction" to a given party based on its (Euclidean) distance to that party; accordingly, a voter's attraction to Party A diminishes as the distance between the voter and the Party A grows. For some voters, the attraction to a given party may be so weak that they will not vote at all. The model assumes, in this regard, that voters will not wait in line at the polls if their attraction to parties or candidates is lower than their perceived "cost of voting."

Following the gradient rule, on each iteration a party examines the points immediately surrounding it and moves to an adjacent point if the move puts the party in a position to receive more votes. In doing so, the party must "imagine" its prospective payoffs in the "local" strategy space. The parties' respective payoffs are a function of the number of likely voters that will support them, and the parties may incrementally change their strategies in order to garner more support (i.e., to increase their payoff by moving locally according to the gradient rule); a change in the strategy (or location) of one party will affect the strategies of its counterpart. Do these assumptions lead to different outcomes than in Anthony Downs' "median voter model?" Under what conditions do parties converge and what parameters give rise non-convergent outcomes?

A two-dimensional voting model developed with Netlogo illustrates that under some conditions, one gets convergence of the two platforms (essentially to a bivariate median voter), but under other conditions the two parties remain far apart in steady state. A key parameter is the cost of voting: the degree to which citizens are less likely to vote when they see little difference between the parties, or see both as very distant from their own preferred position in issue space. When the cost of voting is zero, parties converge to the highest density of voters, an outcome that is consistent with the Downsian median voter model. As the cost of voting increases, however, parties do not converge. The distance between parties when they reach a steady state is a function of the cost parameter. This dynamic approach seems to avoid the indeterminacy that plagues equilibrium models.

7 Conclusion

As our three examples show, landscape dynamics combines the modeling ease of agent-based modeling with the powerful analytical tools of global analysis. Many more applications may be expected in the near future, and the scope of the complex dynamical systems coming under this approach may increase with the growing capability of computer hardware and software. A feature of NetLogo called HubNet provides a platform for experiments combining human subjects and robotic agents, a kind of science beyond the scope of mathematical modeling alone. And feedback from the frontiers of science are influencing the further develeopment of ABM tools. This is a golden age for the social sciences.

8 References

8.1 Articles and Books

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8.2 Websites

Ralph Abraham, http://www.ralph-abraham.org

Dan Friedman, http://leeps.ucsc.edu/people/Friedman

Pablo Viotti, http://www.viotti.com

Landscape Dynamics, http://www.vismath.org/research/landscapedyn

Learning and Experimental Economics Projects, http://leeps.ucsc.edu/projects

NetLogo, http://ccl.northwestern.edu/netlogo

9 Figures

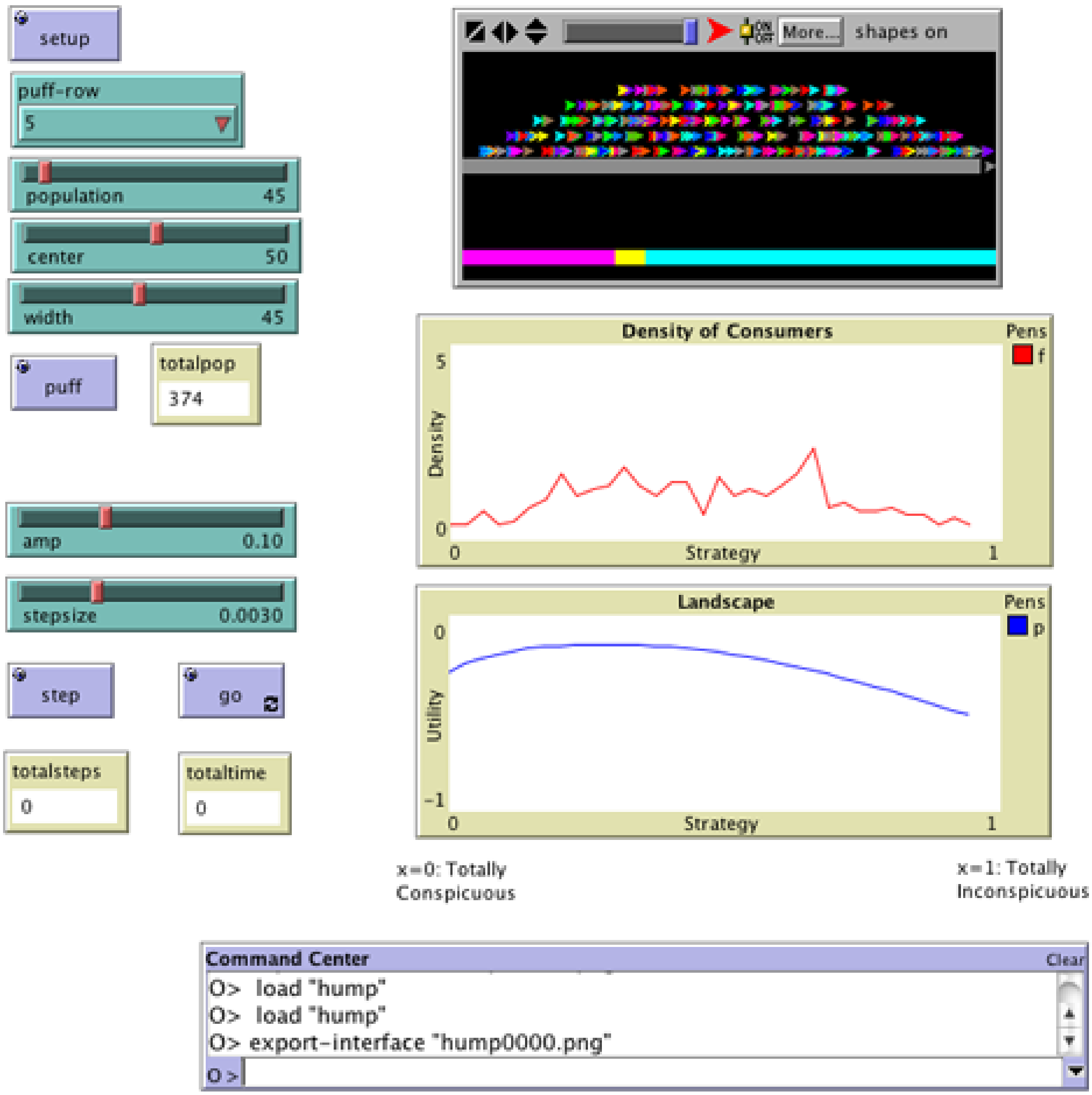

Figure 1: Interface of the consoicuous consumption model, Veblen 5.2

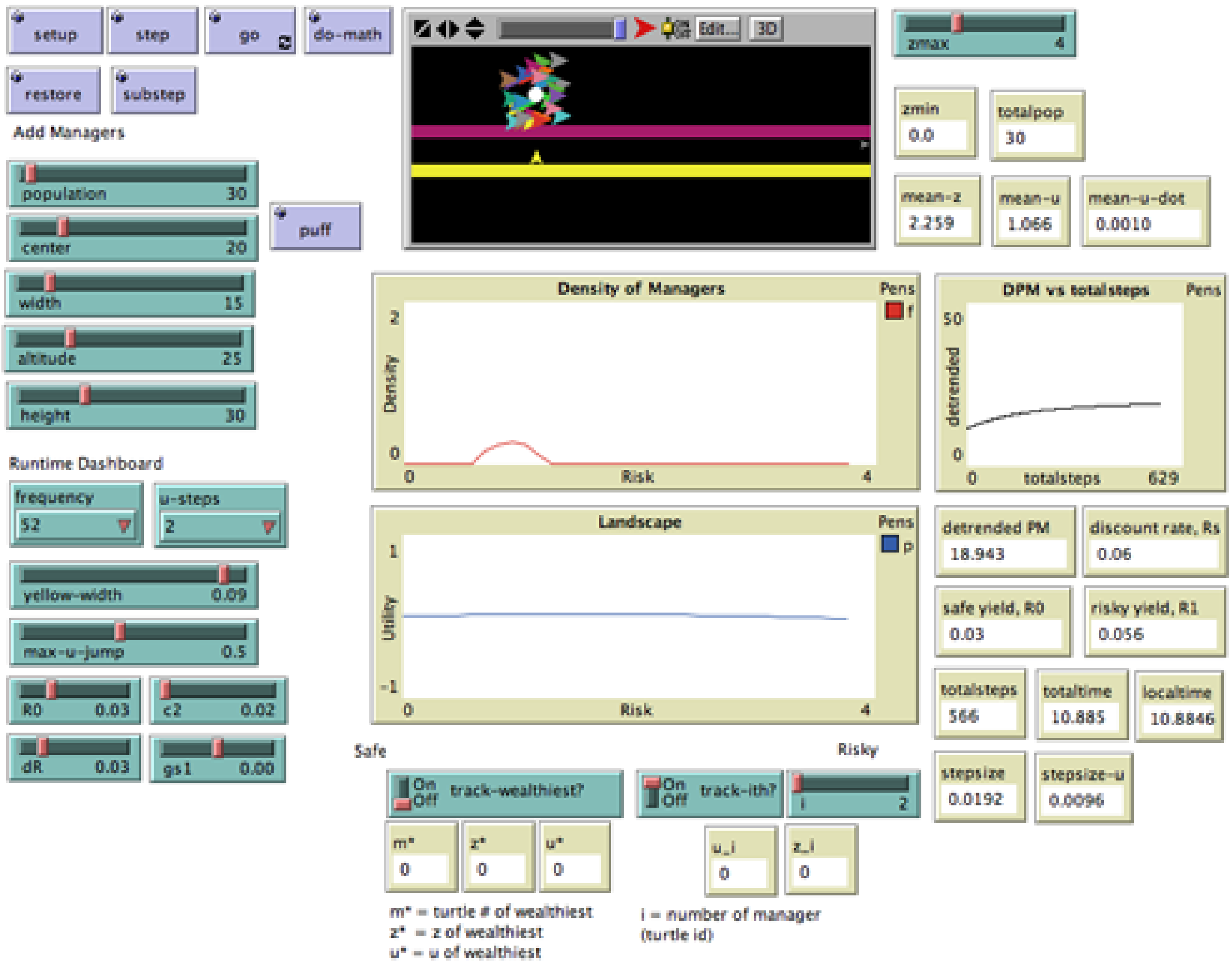

Figure 2: Interface of financil market model, Market 8.0.

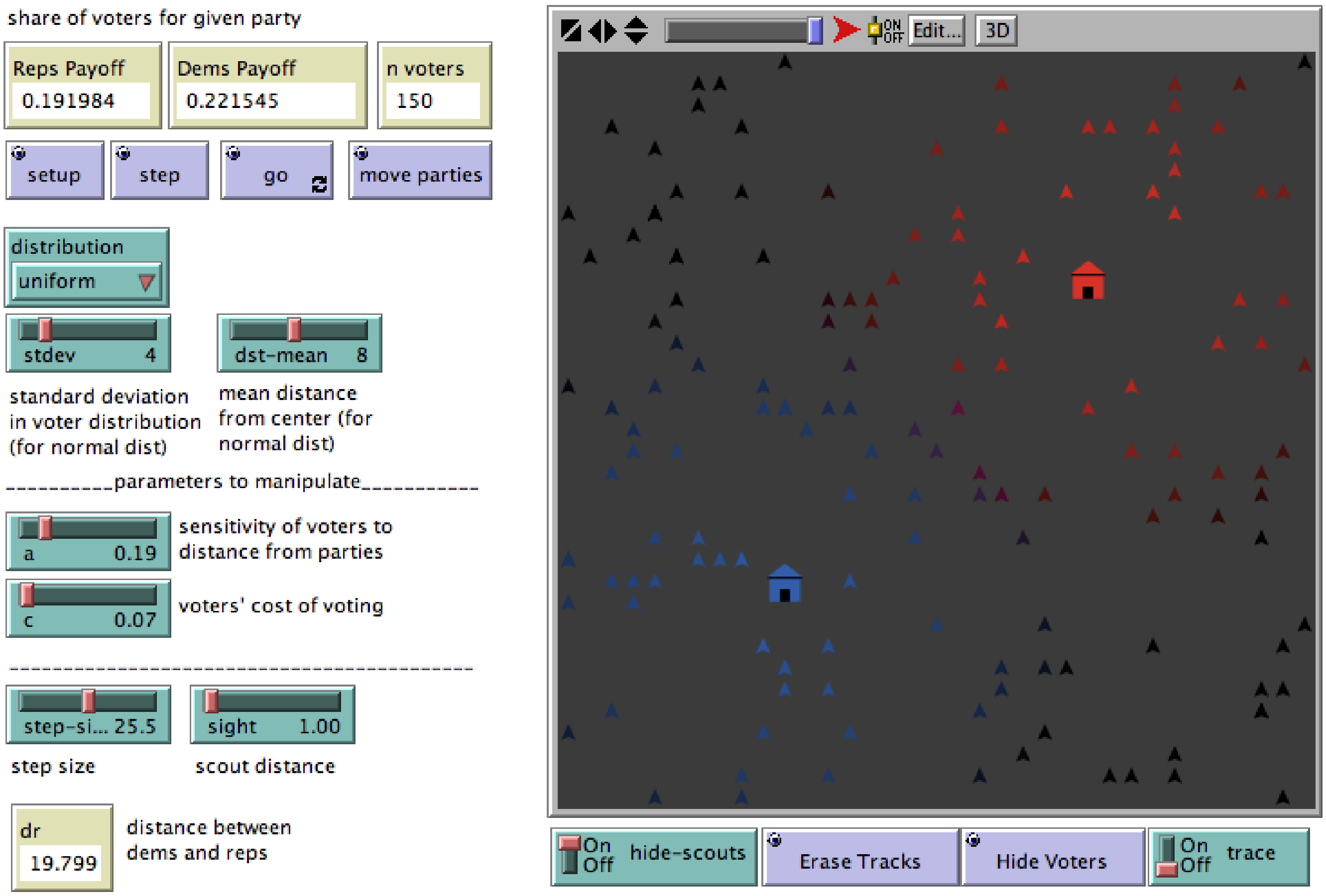

Figure 3: Interface of voting model, Downs 03-07.