Golden Ratio

GRADE: 6-8
PERIODS: 1

STANDARDS:

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Location: unknown

Students explore the Fibonacci sequence, examine how the ratio of two consecutive Fibonnacci numbers creates the Golden Ratio, and identify real-life examples of the Golden Ratio.

Instructional Plan | Objectives + Standards | Materials | Assessments + Extensions | Questions + Reflection | Related Resources | Print All

Begin class with the Fibonacci Rabbits Activity Sheet.

Fibonacci Activity Sheet

The first page shows the problem posed by Fibonacci 800 years ago. The second page shows one strategy for solving the problem. You may wish to distribute the first page only.

> A pair of rabbits cannot bear young until they are two months old. But once a pair reaches maturity, they will produce one new pair of rabbits each month.
>
> If you start with one pair of new-born rabbits, how many pairs of rabbits will you have at the beginning of each month thereafter?

Use the second page of the activity sheet with rabbit pictures, photocopied as a handout for each student to track their work. In the first month, you have 1 pair. In the second month, there is still only 1 pair since they aren't old enough to reproduce. In the third month, the first pair reproduces, so there will be 2 pairs. In the fourth month, only the first pair is old enough to reproduce, so there will be 3 pairs. By the fifth month, the original and the next pair are able to reproduce, so there will now be 5 pairs. By the sixth month, one more additional pair is old enough to reproduce, so we will be adding 3 more pairs.

Students should take about 5-10 minutes to individually think about ways they could solve this problem and record those strategies on the first page of the activity sheet. Then, they can meet with a partner to discuss these strategies.

This leads to the Fibonacci Sequence of 1, 1, 2, 3, 5, 8, 13, 21, 34, 55.... The students should keep the chart up to date with these solutions. Use the second page of the activity sheet.

A physical model of the pairs will also work, keeping track of which pairs are old enough to reproduce.

Have students verbalize the pattern (add previous two numbers to create the next number). Once students see that the pattern is adding the previous number to each number—for example, the number after 5 is 8, because 3 + 5 = 8 —have them predict the next several numbers in the pattern.

Next, students should open a new spreadsheet file, using Microsoft Excel® or another program.

Label the first column Item Number; then, in A2, enter "1." In A3, enter "=A2+1." Select the cell that contains the formula, and fill down (using the edit pull-down menu). Title the next column Fibonacci Number. In B1, enter "1." In B2, write "1." In B3, enter the formula "=B1+B2." The computer will add the numbers from B1 and B2. Again, select the cell with the formula, and fill down (using the edit pull-down menu). Students will enjoy seeing how many Fibonacci numbers can be generated just by entering just one formula! (Note: *The width of the column will determine how many digits will be displayed before showing scientific notation.*)

Have students make a scatterplot of the data with the spreadsheet. Then, have them describe the shape. [At first it increases very slowly; then, it increases very quickly.] Have students determine if this is linear or non-linear. [Non-linear. There is no constant rate of change. The rate of change, or differences between terms, is in the Fibonacci sequence!]

Fibonacci numbers can be found in many places. Use the Fibonacci in Nature Overhead.

Fibonacci in Nature Overhead

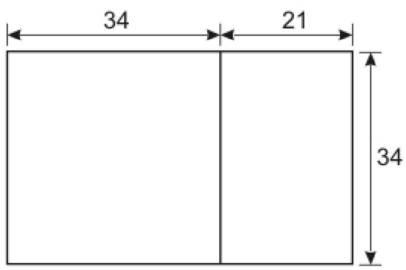
In nature, many plants are in the Fibonacci sequence. The Colorado state flower, the Columbine, has 5 petals. The black-eyed Susan has 13. As a demonstration, you can also cut open an apple and count the number of seeds, or you may count the sections in a sliced lemon.

Measure the distance from the floor to one's waist (navel), and then measure from the navel to the top of the head. What is the ratio of these measurements? Similarly, find the ratio of the distance from the neck to the top of the head to the distance from the neck to the navel; then, find the ratio of the distance from the knee to the floor to the distance from the navel to the knee. How are these ratios related? In an adult, these ratios are approximately equal to the Golden Ratio (as discussed below). For students in varying stages of growth, they may not hold true. Students can work in pairs to help measure each other and record the measurements on a piece of paper.

Next, have students return to the spreadsheet, and find the ratio of one Fibonacci number to the previous Fibonacci number. That is, have students divide one Fibonacci number by the Fibonacci number in the cell below. In C2, have students enter the formula "=B2/B1." Once again, have students fill down, and notice that all values converge to 1.618! This is called the Golden Ratio. Why is it special? Next, have the students, in D2, enter the formula "=B1/B2," and fill down, to find the ratio of a Fibonacci number divided by the next Fibonacci number. This will give the reciprocal of the previous ratio. Yet, it converges on 0.618! The Golden Ratio is the only number that differs from its reciprocal by 1.

Rectangles built in the Golden Ratio are said to be pleasing to the eye. Have students measure the length of a switch plate cover and divide it by the width. Often, the ratio is 1.618 : 1. Do the same with a credit card (or student ID card). The ratio will likely be 1.618 : 1 each time. In ancient times, many famous buildings—such as the Parthenon—were built to these dimensions. As time permits, or as an extension, students can research other objects which appear in the golden ratio.

Using Geometer's Sketchpad, or a similar geometry tool, have students draw rectangles that are in the Golden Ratio, or draw rectangles on graph paper with sides of sequential Fibonacci numbers. What is the ratio of the sides? Examine all of the rectangles. How are they all related? [They are all similar, meaning that the sides are in proportion; they are enlargements of one other. Notice that the ratio of the side lengths of the two rectangles below are 34/21 ≈ 1.619048, and 55/34 ≈ 1.617647. As the Fibonacci numbers get larger, the ratio gets closer and closer to the Golden Ratio.]



Have students explore where the golden rectangle is used. Artists often divide their canvas into a rectangle and a square. This is called the Golden Rectangle. Find objects and paintings that contain the Golden Rectangle. Compare them with objects that are in other ratios. How are they different? Which is more pleasing to the eye? [If a canvas is divided into the Golden Rectangle, the eye is drawn to the line at the right, and it often is the focal point of a painting.]

- Fibonacci Rabbits Activity Sheet
- Fibonacci in Nature Overhead
- Computer Spreadsheet Program, such as Microsoft Excel®
- Dynamic Geometry Software, such as Geometers Sketchpad®
- Measuring tape or rulers
- Calculators
- Apple, lemon (optional)

Assessments

1. As students are creating their spreadsheets and scatterplots, the teacher should be assessing student work by circulating throughout the classroom.
2. As students identify examples of the Golden Ratio in their world, the teacher should listen to student responses to get a sense of their understanding of what a Golden Ratio is.
3. If so desired, students could write their responses to the *Questions for Students* (above) and the teacher could collect the written responses as another form of assessment.

Extensions

1. Examine the Golden Spiral. Each stage will show the square and the rectangles referred to in the Sketchpad activtiy. On a sheet of graph paper, have students draw a square in the center with side length

1, the first Fibonacci Number. Next to it, draw another with side length 1, the second Fibonacci number. Above these two, draw a square with side length 2. Notice you can now draw a square with side length 3 next to it, each time, adding a square with side length the next Fibonacci number. Keep this going until you run out of paper. The rectangles themselves are intriguing, as they also show the sums of the previous two numbers. But if you draw in the diagonal in the first square, and then keep it going into the next and next again, you will get the spiral, known as the golden spiral, that resembles a nautilus shell!
2. Students can find examples of spirals in pine cones, sunflowers, and pineapples.

Questions for Students

1. How do we create the Fibonacci Sequence?

[Start with 1, then 1, and then add the two previous numbers to get the next number in the sequence.]

2. How is a spreadsheet used to create the Fibonacci Sequence?

[The program extends the pattern using the fill down feature.]

3. What happens when you divide one Fibonacci Number by the previous or next Fibonacci Number?

[You get the Golden Ratio or its reciprocal.]

4. How are golden rectangles related?

[They share the same ratio of length and width.]

5. Why was the Golden Ratio used in buildings?

[It was the most visually appealing to the eye.]

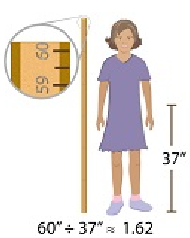
Teacher Reflection

- As students were creating their spreadsheet file, did they demonstrate an understanding of why the file was being set up a certain way (e.g. how the formulas worked)? Did you have to adjust your teaching to increase student understanding of this application?
- Did the problems posed in this lesson (e.g. the Rabbits problem, finding examples of the Golden Ratio in the world around us) provide sufficient motivation for the students, or did you have to adjust the lesson to engage all students?
- How did this lesson address auditory, tactile, and visual learning styles?

A Ratio that Glitters: Exploring the Golden Rectangle

6-8, 9-12

In this lesson, students will develop an understanding of the Fibonacci Sequence (and its connection to Golden Rectangles), Golden Ratio, Golden Rectangle, and the term *ratio* (as it applies to rectangles). Students will use tools and construction techniques to demonstrate geometry prowess and be able to observe the Golden Rectangle in nature and in the classroom.



The Golden Ratio

6-8

Students learn about ratios, including the “Golden Ratio”, a ratio of length to width that can be found in art, architecture, and nature. Students examine different ratios to determine whether the Golden Ratio can be found in the human body.

Learning Objectives

Students will:

- Discover the Fibonacci Sequence with the rabbit problem
- Use a spreadsheet to examine the Fibonacci Sequence
- Graph the Fibonacci numbers in a scatterplot
- Examine similar rectangles
- Recognize the Golden Ratio in nature, architecture, and art

NCTM Standards and Expectations

- Represent and analyze patterns and functions, using words, tables, and graphs.
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Common Core State Standards – Mathematics

Grade 8, Geometry

- CCSS.Math.Content.8.G.A.4
  Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Grade 8, Stats & Probability

- CCSS.Math.Content.8.SP.A.1
  Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.