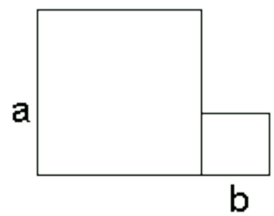
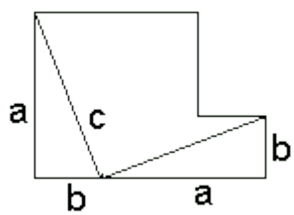
Pythagorean Proofs: $c^2 = a^2 + b^2$

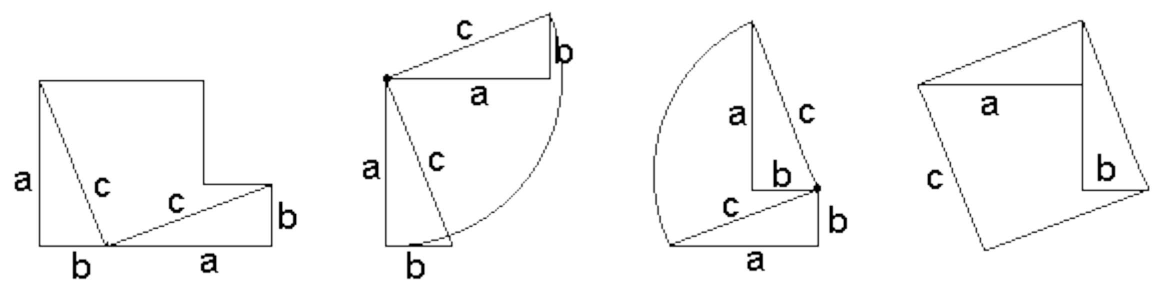
Proof #1



Begin by cutting out two squares whose sides are congruent to the legs of a right triangle. The squares have sides of length **a** and **b**. Tape them together.



Using the length a and b, draw the triangles illustrated. Label the third side **c**. Note that the segment common to the two squares has been removed. At this point we therefore have two triangles and a strange looking shape.



Cut the triangles and lay all the pieces flat on the table. As a last step, rotate each triangle 90° about its <u>top</u> vertex. The right one is rotated clockwise whereas the left triangle is rotated counterclockwise. "Obviously" the resulting shape is a square with the side **c** and area $\mathbf{c^2}$. Justify that, indeed, the resulting shape is a square.

How is this a proof of the Pythagorean Theorem?

There are many proofs of the Pythagorean Theorem. These proofs are taken from http://cut-the-knot.org/pythagoras/ *where you can find an additional 92 proofs, many with links to interactive applets.*

Proof #2

Use grid paper to cut out 8 congruent right triangles with sides **a**, **b** and **c**. Also cut out three squares – one with a side length of **a**, one with a side length of **b**, and one with a side length of **c**.

With four of the triangles, and the two smaller squares, compose a larger square with a side length of **(a+b).** Show what you did with a sketch. Justify that this composition is, indeed, a square.

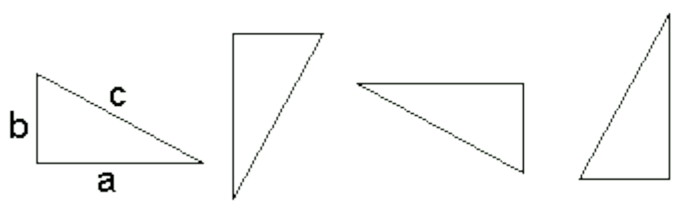
With the remaining pieces (4 triangles and a c-square) compose another larger square also with a side length of **(a+b).** Show what you did with a sketch. Justify that this composition is, indeed, a square.

Using these two composed squares, act out a justification for the Pythagorean Theorem.
Explain what you did to justify that $a^2 + b^2 = c^2$. Use drawings in your explanation.

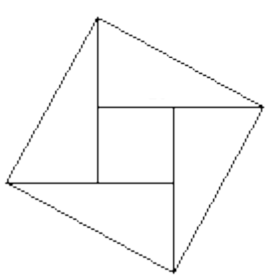
Write an algebraic justification that symbolizes what you acted out.

Proof #3

Folding a paper into quarters will allow you to easily cut four congruent right triangles. Label the legs **a** and **b** and the hypotenuse **c** on each triangle.



Now start with four copies of the same triangle placed in a pile. Rotate the top one 90°; the next one, 180°; and the third one, 270°.



Next, translate the four triangles to form a square with a square hole in the center. How do you know the large figure is a square? How do you know the hole is also a square?

What is the side length of the large square? What is the side length of the small square?

The large square is composed of five shapes. What is the area of each of these shapes?

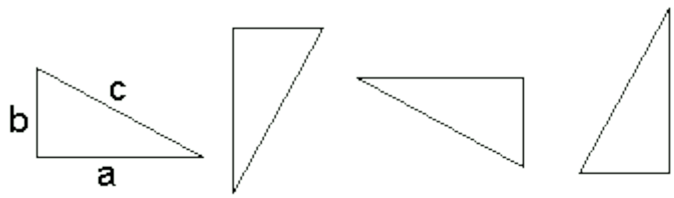
What is the area of the large square based on its side length?

What is another way to express the area of the large square based on the area of the smaller shapes? Can this expression be simplified?

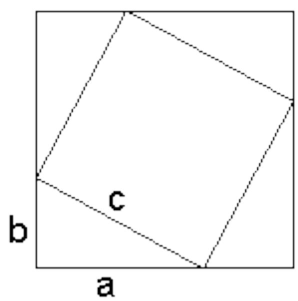
How is this a justification of the Pythagorean Theorem?

Proof #4

Folding a paper into quarters will allow you to easily cut four congruent right triangles. Label the legs **a** and **b** and the hypotenuse **c** on each triangle.



Now start with four congruent right triangles placed in a pile. Rotate the top one 90°; the next one, 180°; and the third one, 270°.



Next, translate the four triangles to form a square with a square hole in the center. How do you know the large figure is a square? How do you know the hole is also a square?

What is the side length of the small square? What is the side length of the large square?

The large square is composed of five shapes. What is the area of each of these shapes?

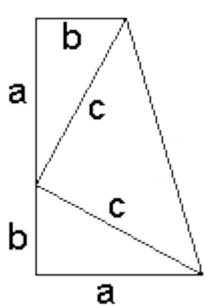
What is the area of the large square based on its side length?

What is another way to express the area of the large square based on the area of the smaller shapes? Can this expression be simplified?

How is this a justification of the Pythagorean Theorem?

Proof #5

This proof, discovered by President J. A. Garfield in 1876 [Pappas], is a variation on the previous one. But this time no squares are drawn at all. The key now is the formula for the area of a trapezoid - *half the sum of the bases times the altitude.* Looking at the picture another way, the area of this figure also can be computed as the sum of the areas of three right triangles.



First, justify that the larger figure is a trapezoid. Next, justify that the three triangles are right triangles. Now right an algebraic equation for the area of the trapezoid equal to the sum of the areas of three triangles. Solve this equation for $\mathbf{c}^2$.