NARROW CORRIDOR

Teacher's Guide — Getting Started

Lay Chin Tan
Singapore

Purpose

In this two-day lesson, students are asked to determine whether large, long, and bulky objects fit around the corner of a narrow corridor.

The objective of this lesson is to apply the concept of turning points (maximum or minimum points) and the Pythagorean Theorem to determine the longest object that can go around the corner of a corridor.

Prerequisites

Students should know how to draw and interpret graphs and should know how to identify the maximum and minimum points of a graph. Prior knowledge of Pythagorean Theorem is required.

Materials

Required: Ruler (metric).
Suggested: A graphing calculator or other graphing utility.
Optional: None.

Worksheet 1 Guide

The first four pages of the lesson constitute the first day's work in which students are introduced to the problem of moving a sofa, but then asked to investigate a similar but simpler problem. Instead of a sofa, which is a three-dimensional object, they are asked to explore the case where a plumber tries to carry a long pipe around a corner. Since no prior knowledge in differentiation is necessary, students are expected to use graphing tools to sketch the graph of the mathematical expression that they have formulated. They are then required to interpret the graph(s) and draw a conclusion. This activity can be modified to incorporate differentiation to find the minimum value of a function.

Worksheet 2 Guide

The last two pages of the lesson constitute the second day's work in which students use the results obtained in the earlier class to model the original problem. Different corridor shapes are introduced to incorporate real-world variations within their model.

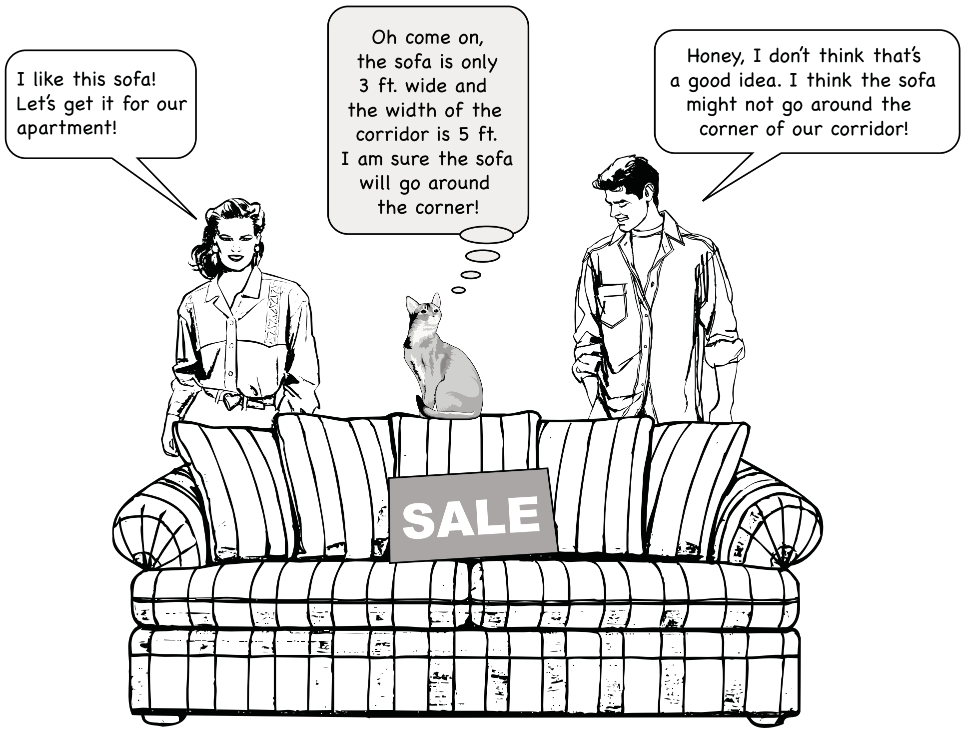
CCSSM Addressed

A-CED.1: Create equations and inequalities in one variable and use them to solve problems.
A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinates axes with labels and scales.
F-IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-BF.1: Write a function that describes a relationship between two quantities.
G-SRT.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

NARROW CORRIDOR

Student Name:________________________________ Date:______________

George and Linda wanted to buy a sofa for their new apartment at a sale. Linda saw a sofa that she really liked. But George thought otherwise.



The sofa is 3 feet wide, 9.5 feet long, and 3 feet high. Figure 1 shows the floor plan of the corridor that leads to George and Linda's new apartment. In addition, the ceiling is 9 feet above the floor.

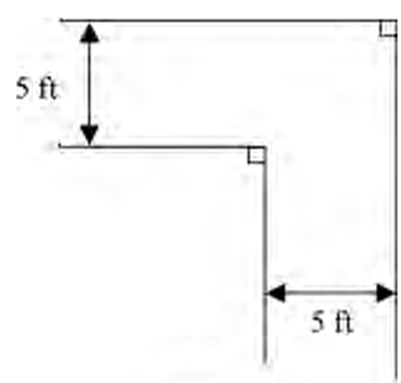


Figure 1

Leading Question

If George and Linda buy the sofa, will they be able to move it into their apartment?

NARROW CORRIDOR

Student Name:______________________________________ Date:__________________

Before modeling with a sofa, think of a similar problem in which a plumber tries to carry a long pipe horizontally around the corner of the corridor. You may assume that the width of the pipe is negligible. If the pipe is too long, it will be stuck at the corner as shown in Figure 2.

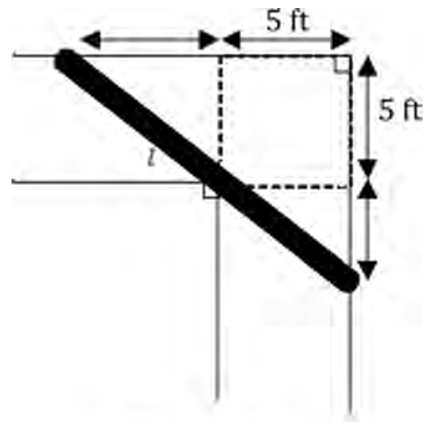


Figure 2

1. Investigate the relationship between l, y, and x in Figure 3 on the next page. Complete the following table by measuring l and y with a ruler for different values of x.

$a =$ ______________ cm.

x cm	y cm	l cm
10		
9		
8		
7		
6		
5		
4		
3		
2		
1		

NARROW CORRIDOR

Student Name:______________________________________ Date:__________________

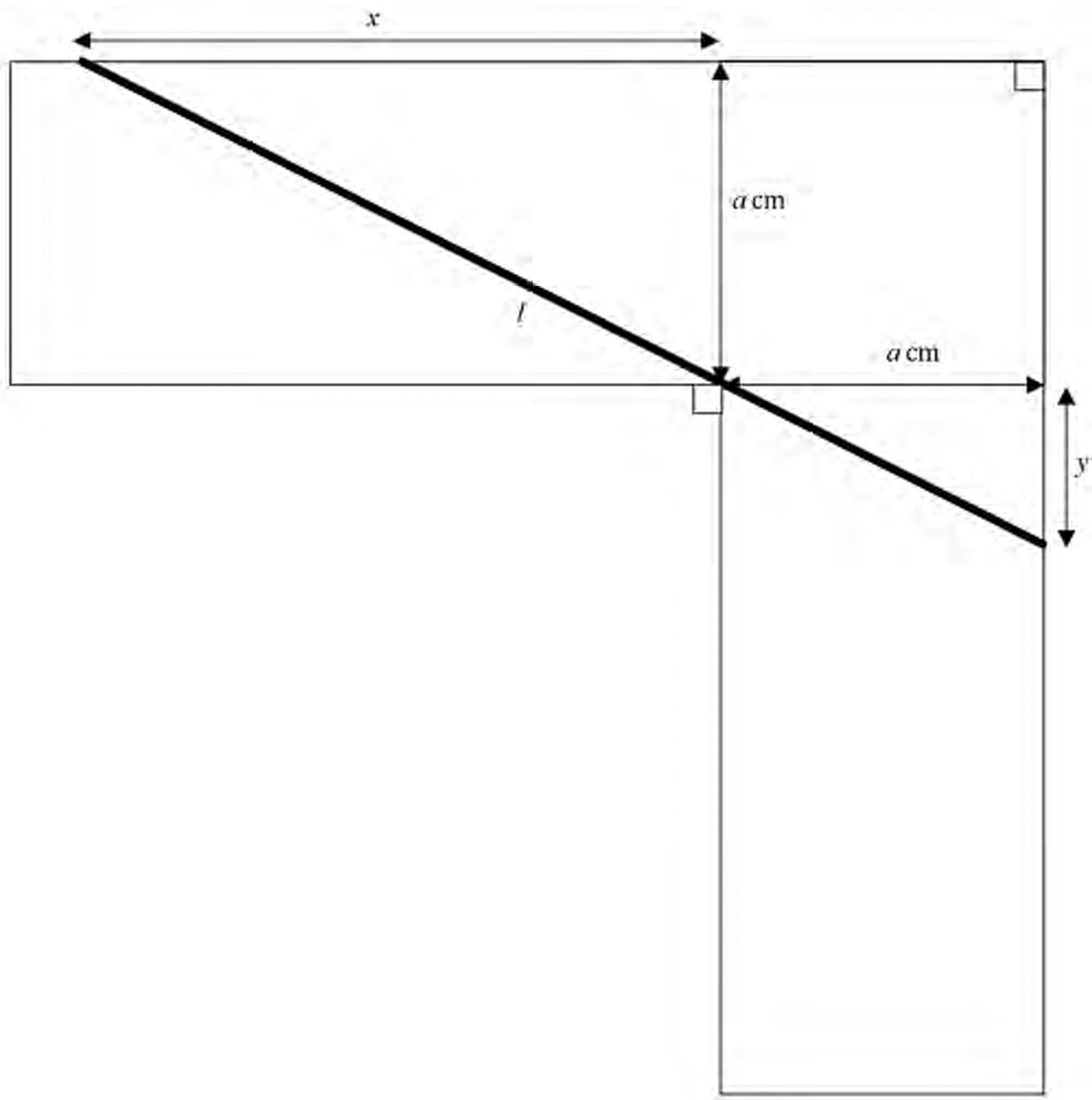


Figure 3

2. Use a graphing calculator to draw the scatter plot of l against x. What do you observe?

NARROW CORRIDOR

Student Name:____ Date:___________________

3. Write an algebraic expression for l in terms of x.

4. With the help of a graphing calculator, sketch the graph of the equation found in question 3 for $1 \leq x \leq 10$. What do you observe? Does the graph fit the scatter plot in question 2?

5. From the graph found in question 4, what is the length of the longest pipe that can go around the corner of the corridor horizontally in Figure 2?

6. If it is not necessary for the plumber to carry the pipe *horizontally*, do you still think the answer obtained in question 5 is the length of the longest pipe that can go around the corner? Justify your answer.

NARROW CORRIDOR

Student Name:____ Date:___________________

Use your previous results to help solve the original question of moving a sofa around the corner of a corridor.

7. What is the length of the longest sofa with a width of 3 feet that can go around the corner of the corridor horizontally?

8. If the movers are allowed to tilt the sofa while moving it, what is the length of the longest sofa that can go around the corner of the corridor? Do you think George and Linda should buy the sofa?

NARROW CORRIDOR

Student Name:____ Date:____________________

9. If the corner of the corridor makes an angle of 120˚ instead of a right angle as shown in Figure 4, what is the length of the longest sofa with a width of 3 feet and a height of 3 feet that can go around the corner? Should George and Linda buy the sofa in this case?

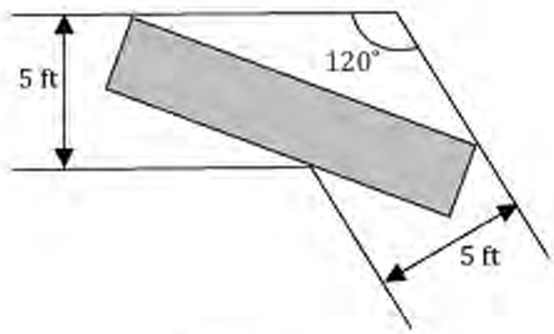


Figure 4

10. Suppose George's and Linda's apartment is along the corridor as shown in Figure 5 and the width of the door is 4 feet and its height is 8 feet. Will the longest possible sofa found in questions 7 and 8 be able to fit through the door?

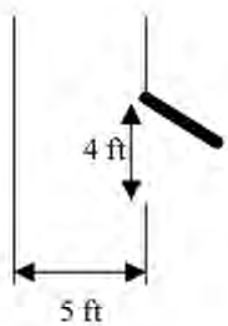


Figure 5

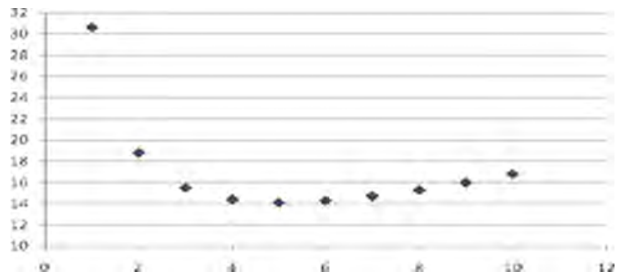
NARROW CORRIDOR

Teacher's Guide — Possible Solutions

The solutions shown represent only some possible solution methods. Please evaluate students' solution methods on the basis of mathematical validity.

x cm	y cm	l cm
10	2.5	16.8
9	2.8	16.0
8	3.1	15.3
7	3.6	14.7
6	4.4	14.3
5	5.0	14.1
4	6.3	14.4
3	8.3	15.5
2	12.5	18.8
1	25	30.6

1. Students should obtain a value of approximately $a = 5$ cm.
2. A scatter plot on their graphing calculator should look similar to the one pictured. They should conclude that there exists a minimum value of l as x varies. In other words, there exists the shortest pipe that will be stuck at the corner of the corridor and it appears to occur when $x = 5$.

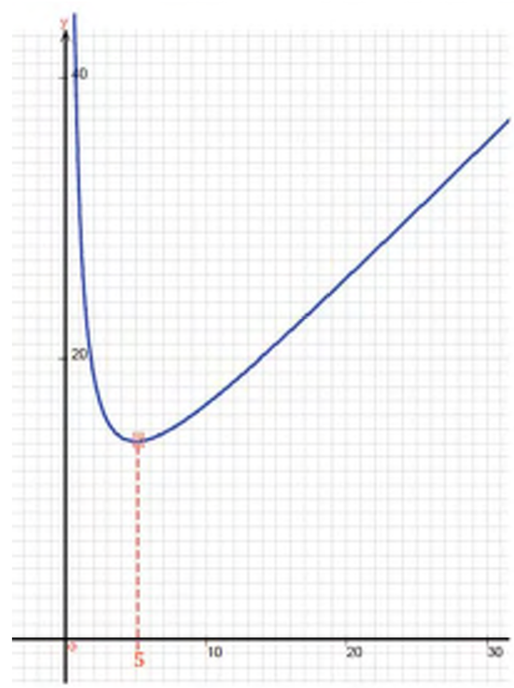


3. By the Pythagorean Theorem, $l^2 = (x + 5)^2 + (y + 5)^2$. By similar triangles, $\frac{y}{5} = \frac{5}{x} \rightarrow y = \frac{25}{x}$.

   Therefore, $l^2 = (x + 5)^2 \left(\frac{25}{x} + 5\right)^2$ which yields $l = \sqrt{(x+5)^2 + \left(\frac{25}{x}+5\right)^2}$

4. The graph of l against x for $x > 0$ has a minimum point at $x = 5$.

   When $x = 5$, $l = \sqrt{10^2 + (5+5)^2} = 10\sqrt{2}$ When the graph is superimposed on the scatter plot in question 2, the graph should fit the scatter plot well. The solution can also be obtained by using trigonometric functions.



NARROW CORRIDOR

Teacher's Guide — Possible Solutions

5. From question 4, the length of the longest pipe that can go around the corner is $10\sqrt{2}$ ft.
6. Using the Pythagorean Theorem, the length of the longest pipe that can go around the corner is 16.8 ft.
7. The length of the longest sofa that can go around the corner horizontally is 8.14 ft. So the sofa, which is 8.5 ft. long, will not be able to go around the corridor horizontally.
8. The length of the longest sofa that can go around the corner of the corridor is 9.18 ft. George and Linda should not buy the sofa.
9. The length of the longest sofa that can go around the corner of the corridor is 10.2 ft.
10. If the sofa is moved horizontally, the length of the longest sofa that can pass through the door is 6.67 ft. The length of the longest sofa that can go around the corner of the corridor is 7.60 ft and so the sofa found in questions 7 and 8 will not be able to pass through the door.

NARROW CORRIDOR

Teacher's Guide — Extending the Model

The length of the longest pipe that would go around the 90° corner was computed by using the Pythagorean Theorem. The horizontal distance was $x + 5$, and the vertical distance was $(25/x) + 5$. Hence:

$$l = \sqrt{(x-5)^2 + (\frac{25}{x} - 5)^2}.$$

The length l could have been computed in a different way. The pipe can be thought of as consisting of two pieces, one to the left of the point where it is up against the interior wall, and one to the right of that point. The piece to the left has length

$$\sqrt{x^2+25}$$

while the piece to the right has length

$$\sqrt{(\frac{25}{x})^2+25}.$$

Thus, the length can also be written as

$$l = \sqrt{x^2+25} + \sqrt{(\frac{25}{x})^2+25}.$$

Perfectly true, but perhaps unexpected. It is unusual in high school algebra for the sum of two such differently looking square roots to equal yet another different single square root. You can see why it is true in our model, but why is it true algebraically?

The question about going around the 120° corner leads to another interesting problem. If we have an obtuse triangle with sides of length a and b on either side of the 120° angle, how long is the side opposite the 120° angle? By the law of cosines, we get that $c^2 = a^2 - 2ab \cos 120° + b^2 = a^2 + ab + b^2$. So it is natural to ask the question, "What corresponds to Pythagorean triples in a 120° triangle?" Are there integers a, b, and c such that $a^2 + ab + b^2 = c^2$? Well, $a = 5$ and $b = 3$ yield $c = 7$, so it can certainly happen. Other examples are (7, 8, 13) and (7, 33, 37). [No, it is not true that all solutions involve 7. There is (5, 16, 19).] Here is a general formula for solutions: pick integers m and n, and let

$$a = 3n^2 + 2mn$$

$$b = 2mn + n^2$$

$$c = 3n^2 + 3mn + n^2.$$

See the following reference for an application of this bit of mathematics in the context of high-speed photography.

Reference

Gilbert, E.N. (1963)., Masks to pack circles densely, *J.SMPTE 72*, 606-608