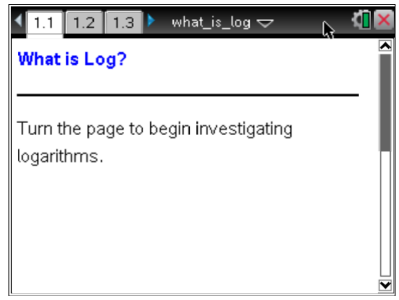
Math Objectives

- Students will recognize the function $g(x) = \log_b(x)$ as the inverse of $f(x) = b^x$ where $b > 0$ and $b \neq 1$.
- Students will apply this inverse relationship and solve simple logarithmic equations.
- Students will look for and make use of structure (CCSS Mathematical Practice).



Vocabulary

- exponential function
- logarithmic function
- one-to-one function
- inverse function
- domain and range

About the Lesson

- This lesson involves the one-to-one function $f(x) = b^x$. In acknowledging the existence of its inverse, students will:
    - Use the domain and range of $f(x)$ to determine the domain and range of $f^{-1}(x)$.
    - Interpret the graph of $f^{-1}(x)$ as the reflection of $f(x)$ across the line $y = x$.
    - Use this inverse relationship to write an equation for the graph of the inverse.
    - Recognize the logarithmic notation needed to define the inverse function.
    - Use the inputs and outputs of two inverse functions to complete a table.
- As a result, students will:
    - Solve simple logarithmic equations and verify solutions using the corresponding exponential equations.

TI-Nspire™ Navigator™ System

- Use Live Presenter for student demonstrations
- Use Quick Poll to assess students' understanding throughout the activity.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Drag a point on a slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity
What_is_Log_Student.pdf
What_is_Log_Student.doc

TI-Nspire document
What_is_Log.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Then, press ctrl to grab the point and close the hand ( ).

Tech Tip: The pages that have draggable values have been designed to easily allow students to move the point. Instruct students to move the cursor to the open point until they get the open hand ( ) and click the Touchpad or press enter. The point should slowly blink. Then the point can be moved by pressing the directional arrows of the Touchpad.

TI-Nspire Navigator Opportunity: *Quick Polls*
See Note 1 at the end of this lesson.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

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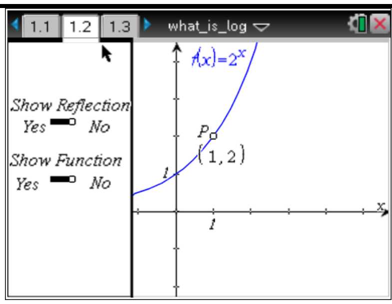

1. The graph of the function $f(x) = 2^x$ is shown.
   a. What are the domain and range of $f(x)$?

      Answer: The domain is $(-\infty,\infty)$ and the range is $(0,\infty)$.

   b. Recall that $f(x) = 2^x$ is a one-to-one function, so it has an inverse reflected over the line $y = x$. What are the domain and range of $f^{-1}(x)$?

      Answer: The domain is $(0,\infty)$ and the range is $(-\infty,\infty)$.

   c. Point P is a point on $f(x)$. Move the Show Reflection slider to *Yes* and then move point P. As you do so, point P' invisibly traces out the graph of $f^{-1}(x)$. Since $f(x)$ can be written as $y = 2^x$, write a corresponding equation for the inverse.

      Answer: $x = 2^y$

Teacher Tip: Point P and P' may not necessarily show the same number of digits, but will round to be the same.

d. The equation $x = 2^y$ cannot be written as a function of *y* in terms of *x* without new notation. Move the Show Function slider to *Yes*. The inverse of $f(x)$ is actually $f^{-1}(x) = \log_2(x)$. In general, $\log_b x = y$ is equivalent to $b^y = x$ for *x* > 0, *b* > 0 and $b \neq 1$. Why do you think *x* and *b* must be greater than 0? Why can *b* not be equal to 1?

Answer: *x* must be greater than 0 because the range of $f(x) = b^x$ is $(0, \infty)$ and thus the domain of $f^{-1}(x) = \log_b(x)$ must be $(0, \infty)$. *b* must be greater than 0 because negative values for *b* will result in negative values for *x*, and *x* has to be greater than 0. *b* cannot be equal to 1 because when *b* = 1, the function is linear, not exponential.

e. Move point *P* so that its coordinates are $(1, 2)$. The point (1, 2) on $f(x) = 2^x$ indicates that $2^1 = 2$. P' has the coordinates $(2, 1)$. The point (2, 1) on $f^{-1}(x) = \log_2(x)$ indicates that $\log_2 2 = 1$. Use this relationship between exponential expressions and logarithmic expressions to complete the following table. (Move point *P* as necessary.)

Answer: See table that follows.

Teacher Tip: Students will not be able to drag point *P* to all possibilities in the table. Encourage them to use the relationships.

P	***P'***	**Exponential Expression**	**Logarithmic Expression**
$(1,2)$	$(2,1)$	$2^1 = 2$	$\log_2 2 = 1$
$(2,4)$	$(4,2)$	$2^2 = 4$	$\log_2 4 = 2$
$(3,8)$	$(8,3)$	$2^3 = 8$	$\log_2 8 = 3$
$(0,1)$	$(1,0)$	$2^0 = 1$	$\log_2 1 = 0$
$\left(-1,\frac{1}{2}\right)$	$\left(\frac{1}{2},-1\right)$	$2^{-1} = \frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
$\left(-2,\frac{1}{4}\right)$	$\left(\frac{1}{4},-2\right)$	$2^{-2} = \frac{1}{4}$	$\log_2 \frac{1}{4} = -2$
$\left(-3,\frac{1}{8}\right)$	$\left(\frac{1}{8},-3\right)$	$2^{-3} = \frac{1}{8}$	$\log_2 \frac{1}{8} = -3$

Teacher Tip: Students may need to be reminded that $2^{-n}=\dfrac{1}{2^n}$ and thus $\log_2\dfrac{1}{2^n}=-n$.

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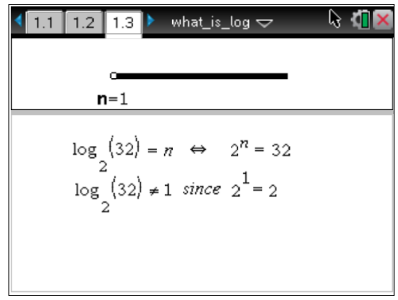

2. Solve the logarithmic equation $\log_2 32=y$ using the patterns from question 1. Then, use the slider to change the *n*-value to solve the logarithmic equation. How does the exponential equation verify your result?

   Answer: $n=5$ since $2^5=32$.

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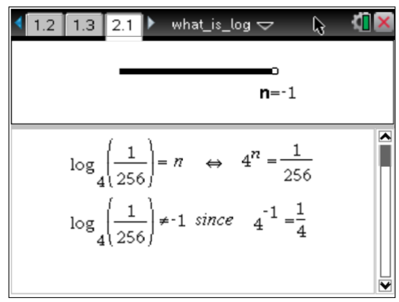

3. Solve the equation $\log_4\dfrac{1}{256}=y$. Then, use the slider to change the *n*-value to solve the logarithmic equation. How does the exponential equation verify your result?

   Answer: $n=-4$ since $4^{-4}=\dfrac{1}{256}$.

4. Maya solved the logarithmic equation $\log_4 16=y$. She says the answer is 4 since 4 times 4 is 16. Is her answer correct? Why or why not?

   Answer: Maya is not correct. The logarithmic equation $\log_4 16=y$ is equivalent to the exponential equation $4^y = 16$. Although $4\cdot 4=16$, the solution to the equation is an exponent and $4^4\neq 16$. The correct solution is $y=2$. Therefore, $\log_4 16=2$.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

5. Alex says that when solving a logarithmic equation in the form $\log_b a=y$, he can rewrite it as $b^a=y$. Is this a good strategy? Why or why not?

   Answer: Alex is not correct. There is an inverse relationship between logarithms and exponentials, but the correct exponential equation is $b^y = a$.

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- For all positive real *b*, where $b \neq 1$, $\log_b x = y$ if and only if $b^y = x$.

Assessment

Determine the value of the following logarithmic expressions and then justify each answer using an exponential expression.

1. $\log_3 27$
2. $\log_5 1$
3. $\log_7 7$
4. $\log_6 \frac{1}{6}$
5. $\log_4 \frac{1}{64}$

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Note 1

Question 1b and 1c, *Quick Poll:* Send an Open Response Quick Poll, asking students to submit their answer to questions 1b and 1c. If students' answers are incorrect, consider taking a Screen Capture. Identify incorrect responses, briefly discussing common misconceptions. Then identify and discuss correct responses.

Note 2

Question 1c, *Live Presenter:* Consider demonstrating or have a student demonstrate how to drag and move point *P* along the graph of the function or to drag the yes/no sliders.

Note 3

Question 4, *Quick Poll:*

Send an Open Response Quick Poll, asking students to submit their answer to question 4.

Note 4

Question 5, *Quick Poll:*

Send an Open Response Quick Poll, asking students to submit their answer to question 5.