APPENDIX

Activity 1

Consider the function $f(x) = 2x + 1$.

1. Create a table of values for the function.

x	$f(x)$

When we discuss additive inverses, if the input is 2, the output is –2; if the input is –2, the output is 2. When we discuss multiplicative inverses, if the input is 4, the output is 1/4, and vice versa. With functions, if we note a function as *f*, we note its inverse (if it too is a function) as f^{-1}.

2. In this function, if the input is 1, the output is 3; that is, $f(1) = 3$. What is the value of $f^{-1}(3)$?

3. Make a new column in the table, name it f^{-1}, and complete the column. (For each *x*, ask yourself this question: If this value is the output from the function, what is the input?)

4. Graph both the original function, with input x and output $f(x)$, and the inverse, with input x and output $f^{-1}(x)$, on the same grid.

   (*a*) What relationship do you notice between the two graphs?
   (*b*) Explain how you could use one graph to find the other; justify your answer.

5. Explain how the characteristics (e.g., domain, range, intercepts, asymptotes, etc.) of this function and its inverse are related. Do you think that this is true for any function and its inverse? Why or why not?

6. What does it mean for an inverse relation of a function to be a function itself?

7. Determine the following values:
   $f(f^{1}(-3))$
   $f(f^{1}(5))$
   $(f^{1}(f(3)))$
   What pattern do you notice? How does the pattern relate to inverse relationships?

8. Determine the following values:
   $f(f^{1}(x))$
   $(f^{1}(f(x)))$
   Again, what pattern do you notice?

Function composition is defined as applying the results of one function to another function. That is, the output of one function becomes the input of a second function. Two widely used notations for function composition, both meaning that function *f* is composed with function *g*, are $f(g(x))$ and $f \circ g(x)$.

9. Use your conjectures from problems 7 and 8 to explain why function composition can be used to verify that two functions are inverses.

Activity 2

Bacterial cells, which are generally spherical, are usually 1 to 5 micrometers in diameter (McDarby 2010–11). However, other cells can grow much larger, with an approximate maximum diameter of 100 micrometers. All cells have a plasma membrane that separates the inside of the cell from its surroundings and helps regulate the passage of materials in and out of the cell. Therefore, the size of this membrane can be quite important in a cell's health.

Let's see how the diameter of a spherical cell influences the cell's surface area.

1. Make a table that lists the diameter and surface area of a variety of cells. (Leave your surface area answers in terms of π.)

2. Write a function rule that expresses the relationship between the radius of the cell and its surface area. What does it mean for the surface area to be a function of the radius?

3. Interchange the columns and create a second table so that surface area is the independent variable and diameter is the dependent variable.

4. Graph the data from the first table. Describe what a graph of the data from the second table would look like. Explain your reasoning.

5. If we wrote a rule (equation) for the relationship in problem 3, how would the new rule relate to the original? Explain your answer.

6. Write a rule (equation) for the data in the second table.

7. Is the new relation in problem 6 also a function? How do you know?

Activity 3

This activity uses your knowledge of exponential functions and their graphs as well as your knowledge of inverse functions to determine the graph of the inverse of an exponential function.

Trace an accurate set of coordinate axes on a piece of wax paper. Center the origin in the middle of the wax paper.

1. Draw and label an accurate graph of $f(x) = 2^x$ on the wax paper. Describe the domain, range, asymptotes, and intercepts for this function. Be sure to label your axes and use the same units on each axis.

2. We are going to create the graph of the inverse of an exponential function.

   (*a*) From your prior knowledge of inverses, determine how the graphs of inverses are related to each other.
   (*b*) What do you expect the graph of the inverse of an exponential function to look like?

Trace the axes (but not the labels) and your exponential graph on a new piece of wax paper. Use your knowledge of the correspondence between the domain and the range of a function and its inverse to label the axes on the second piece of wax paper so that your graph accurately represents the graph of the inverse of your exponential function. (You may label whichever side of the wax paper that you need to obtain the inverse graph of the exponential graph.)

3. Refer to your conjectures in problem 2b. If you were correct, explain how you knew the characteristics of this new graph. If you were not correct, explain how the resulting graph is different from your prediction. Why are they different?