

$$\frac{d^3 y}{dx^3}$$

Question: Determine $\frac{d^3 y}{dx^3}$ at $x=5$ from the following table:

X	2	4	9	13	16	21	29
y	57	1345	66340	402052	1118209	4287844	21242820

Solution:

The divided difference table for the given data is constructed as :

x	y	1 st D.D	2 nd D.D	3 rd D.D	4 th D.D	5 th D.D	6 th D.D
2	57						
4	1345	644					
9	66340	12999	1765				
13	402052	83928	7881	556			
16	1118209	238719	22113	1186	45		
21	4287844	633927	49401	2274	64	1	
29	21242820	2119372	114265	4054	89	1	0

Now using newton's divided difference formula we have

$$y = f(x) = y_0 + (x - x_0)(1^{st} D.Dy_0) + (x - x_0)(x - x_1)(2^{nd} D.Dy_0) + (x - x_0)(x - x_1)(x - x_2)(3^{rd} D.Dy_0) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)(4^{th} D.Dy_0) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)(5^{th} D.Dy_0) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)(x - x_5)(6^{th} D.Dy_0)$$

$$f(x) = 57 + (x - 2)(644) + (x - 2)(x - 4)(1765) + (x - 2)(x - 4)(x - 13)(556) + (x - 2)(x - 4)(x - 13)(x - 16)(45) + (x - 2)(x - 4)(x - 13)(x - 16)(x - 21)(1) + (x - 2)(x - 4)(x - 13)(x - 16)(x - 21)(x - 29)(0)$$

Now by simplification we get

$$f(x) = x^5 - 11x^4 + 106x^3 - 919x^2 + 4014x - 4999$$

So

$$f'(x) = 5x^4 - 44x^3 + 318x^2 - 1838x + 4014$$

$$f''(x) = 20x^3 - 132x^2 + 636x - 1838$$

$$f'''(x) = 60x^2 - 264x + 636$$

$$\frac{d^3 y}{dx^3} \text{ At } x=5$$

$$\frac{d^3 y}{dx^3} = 816(\text{Answer})$$

