FAQ'S

Lecture # 32 Second Fundamental Theorem

Q.1# why we use the second fundamental theorem of calculus?

Answer: The first fundamental theorem of calculus is used to evaluate the definite integral of a continuous function if we can find an anti-derivative for that function, but it does not point out the question of which functions actually have antiderivatives; then at this stage the second fundamental theorem of calculus is used.

The **second fundamental theorem of calculus** states that the derivative of a definite integral with respect to its upper endpoint is its integrand; it allows one to compute the definite integral of a function by using any one of its infinitely many antiderivatives. This part of the theorem has invaluable practical applications, because it markedly simplifies the computation of definite integral.

Q.2# why the function defined by $F(x)=\int_a^x f(t)\,dt$ is not an arbitrary antiderivatives of f ; if $x=a$?

Answer: The function defined by $F(x)=\int_a^x f(t)\,dt$ is not an arbitrary anti derivative of f ; it is the specific anti derivative whose value at $x=a$ is zero

Lecture 41 to 44

Q.3# what is the difference between a sequence and series?

A **sequence** is a number pattern in a definite order following a certain rule.

Examples of sequences:

1) 1, 2, 3, 4, 5, 6, 7 ... *add 1 to the preceding term*

2) 2, 4, 7, 11, 16, 23, 31. *add 2 to the preceding term, add 3 to the next term, etc*

3) 1, 1, 2, 3, 5, 8, 13, 21, 34,... *add the two preceding terms together*

A **sequence** is an ordered list of objects (or events). Like a set, it contains members (also called *elements* or *terms*), and the number of terms (possibly infinite) is called the *length* of the sequence. Unlike a set, order matters, and the exact same elements can appear multiple times at different positions in the sequence.

For example, (C, R, Y) is a sequence of letters that differs from (Y, C, R), as the ordering matters. Sequences can be *finite*, as in this example, or *infinite*, such as the sequence of all even positive integers (2, 4, 6,...).

General sequence terms are usually denoted by:

$\mathbf{a}_1, \mathbf{a}_{2,\ldots}\mathbf{a}_n, \mathbf{a}_{n+1\ldots}$

$\mathbf{a}_1$ 1st term

$\mathbf{a}_2$ 2nd term

$\mathbf{a}_n$ nth term

$\mathbf{a}_{n+1}$ (n+1) term

A **series** is a sum of terms in a sequence. It can be denoted by

$$\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4 + \ldots$$

Using the above sequences, we have the following series:

1) 1 + 2 + 3 + 4 + 5 + 6 + 7 +...

2) 2 + 4 + 7 + 11 + 16 + 23 + 31.

3) 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 +...

Q.4#At what condition a sequence is called monotone?

Answer:A sequence which is either always increasing or always decreasing is called a *monotone* sequence. Note that an ``arbitrary'' sequence is not monotone (it will usually sometimes increase, and sometimes decrease).

Answer: Infinite series are widely used in other quantitative disciplines such as physics and computer science

Q.5#How do we use monotone sequence in real life?

Answer: Monotone sequences do happen in real life. For example, the sequence

$a_1 = 3$ $a_2 = 3.1$ $a_3 = 3.14$ $a_4 = 3.141$ $a_5 = 3.1415...$ is how we often describe the decimal expansion of π. Monotone sequences are important because we can say something useful about them which are not true of more general sequences.

Q.6#When a sequence converges or diverges?

Answer: A sequence **converges** to a limit L if and only if the sequence of even numbered terms and odd numbered terms both converges to L e.g. The sequence $1/2, 1/3, 1/2^2, 1/3^2, 1/2^3, 1/3^3$, ... converges to 0. Note that the even numbered terms and odd numbered terms both converges to 0.(mean as the terms of the sequence increases they become closer to zero, i.e. 1/2 =0.5 , $1/2^2$= 0.25, $1/2^3$= 0.125, 0.0625, 0.03125, 0.015625, 0.0078125,....as the number of term increases it becomes closer to zero).

If both or one of even numbered terms and odd numbered terms are not converges to limit L then the sequence **diverges**.

Q. 7#How do we check the convergent or divergent for series?

Answer: For series we have many methods to check that either the series is convergent or divergent. We have Divergence test, Integral test, convergence of p-Series, Comparison test, Ratio test, the Root test, the Limit comparison Test.

There are a number of methods of determining whether a series converges or diverges

The terms of the sequence $\{a_n\}$ are compared to the sequence of $\{b_n\}$ for all n

Where $0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converge then so $\sum_{n=1}^{\infty} a_n$

Similarly if

For all $0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges then so $\sum_{n=1}^{\infty} a_n$

Ratio Test:

Assume that for all n $a_n > 0$

Suppose that there exist "r" such that $\lim_{x \to \infty} |\frac{a_{n+1}}{a_n}| = r$

If r < 0 then the series converges if r >1 then the series diverge

If r =1 then the series may converge or diverge

Similarly we can apply the other tests such as the Integral test, the Limit comparison test; the Alternating series test to check whether the given series converges or diverges.

Q.8#How do we write an alternating series?

Answer: An **alternating series** is any series, $\sum a_n$, for which the series terms can be written in one of the following two forms.

$$a_n = (-1)^n b_n \qquad b_n \ge 0$$
$$a_n = (-1)^{n+1} b_n \qquad b_n \ge 0$$

There are many other ways to deal with the alternating sign, but they can all be written as one of the two forms above. For instance,

$$(-1)^{n+2} = (-1)^n (-1)^2 = (-1)^n$$
$$(-1)^{n-1} = (-1)^{n+1} (-1)^{-2} = (-1)^{n+1}$$

Q.9#When a series is called a converge conditionally?

Answer:A series $\sum_{n=0}^{\infty} a_n$ is said to **converge conditionally** if $\lim_{m\to\infty} \sum_{n=0}^{m} a_n$ exists and is a finite number (not ∞ or −∞), but $\sum_{n=0}^{\infty} |a_n| = \infty.$

A classical example is given by

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

which converges to In 2 , but is not absolutely convergent.

The simplest examples of conditionally convergent series (including the one above) are thealternating series.

Taylor and MaclauriSeries (Lecture# 45)

Q.10#What is the difference between a Taylor series and a Maclaurin series?

Answer: The **Taylor series** is a representation of a function as an infinite sum of terms calculated from the values of its derivatives at a single point. It may be regarded as the limit of the Taylor polynomials. If the series is centered at zero, the series is also called a **Maclaurin series**.

Q.11#What are the applications of the Taylor series?

Answer:

- Some functions have no antiderivative which can be expressed in terms of familiar functions. This makes evaluating definite integrals of these functions difficult because the Fundamental Theorem of Calculus cannot be used. However, if we have a series representation of a function, we can oftentimes use that to evaluate a definite integral.

Example: Suppose we want to evaluate the definite integral

$$\int_0^1 \sin(x^2)\, dx$$

The integrand has no antiderivative expressible in terms of familiar functions. However, we know how to find its Taylor series: we know that

$$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \ldots$$

Now if we substitute $t = x^2$, we have

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{10!} - \frac{x^{14}}{14!} + \ldots$$

In spite of the fact that we cannot antidifferentiate the function; we can antidifferentiate the Taylor series:

$$\begin{aligned}\int_0^1 \sin(x^2)\, dx &= \int_0^1 (x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \ldots)\, dx \\ &= (\frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!} + \ldots)|_0^1 \\ &= \frac{1}{3} - \frac{1}{7 \cdot 3!} + \frac{1}{11 \cdot 5!} - \frac{1}{15 \cdot 7!} + \ldots\end{aligned}$$

Note that this is an alternating series so we know that it converges. If we add up the first four terms, the pattern becomes clear: the series converges to **0.31026.**

- Sometimes, a Taylor's series can tell us useful information about how a function behaves in an important part of its domain.
- Some differential equations cannot be solved in terms of familiar functions (just as some functions do not have antiderivatives which can be expressed in terms of familiar functions). In this case we use the Taylor's series.

Q.12# How do we find McLaren series of $f(x) = e^x$?

Answer: McLaren series are important in many areas of mathematics. They are often used to *define* functions.

To write down the McLaren series we need to know the value at $x = 0$ of every derivative of the function. This is usually the practical problem that we face in working out Taylor series. In this case it is easy since every derivative of e^x is e^x and this has value 1 at $x = 0$. It turns out that this is actually equal to the value of e^x for any value of *x.* So the McLaren series becomes

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

We often find this given as the *definition* of e^x.

Q.13# what is triangle inequity?

In order to form a triangle, the sum of two shortest lengths must be greater than the longest length. If the sum of two smaller side is less than the largest length, a triangle cannot form.

That is main reason that inequality is called TRIANGLE INEQUALITY

The theorem of "Triangle inequality" states that for a pair of real numbers, the absolute value of sum of two numbers can never exceed than the absolute value of first number plus absolute value of second number. That is for a pair of real number "a" and "b", we have

$| a + b | < = |a| + |b|$

The theorem says that for any pair of real numbers a & b, if we find the absolute values of both the numbers and then add those absolute values as

$|a| + |b|$,

the result may be:

Equal to the absolute value taken after summing those numbers first as . $|(a+b)|$ or $|a+b|$

For Example Let $a =2$ and $b = 3$

$$|2+3|=|2|+|3|$$

Greater than the absolute value taken after summing those numbers first as $|(a+b)|$ or $|a+b|$.

This happens in case both numbers have opposite signs.

For Example Let $a = -2$ and $b = 3$ then

$$| -2 + 3| <| -2|+|3|$$

Application of Definite Integral (Lect # 33-38)

Q.14: Where definite integral is applicable?

Answer: There are many applications of definite integral for example: it is used to find the area bounded by curves, area of a surface of revolution, volume of solids (i.e. volume by slicing and washers as well as cylindrical shell), length of plane curves, work, mass and fluid pressure. We use definite integrals to solve many practical problems.

Q.15: How can we find the volume of a disc when it has some definite thickness?

Answer: When we find the area of disc we suppose that the thickness of the disc is zero and at that time area of the disc is πr^2. This area is also called the surface area of the disc and in this case since disc is in 2 dimensions, so its volume is zero. Now if there is some thickness of the disc say h. Then its volume is simply multiplication of its surface area and thickness that is $\pi h r^2$.

Q.16: Either cylinder can consider as hollow or solid?

Answer: A cylinder refers to a solid bounded by a cylindrical surface and two parallel planes. Hollow cylinder is named as cylindrical surface.

Q.17: How do we calculate the volume using shells?

Answer:: A shell is a hollow tube. To calculate volume using shells, we take shells of graduated radii and fill the solid of revolution with them. We only want the volume of the material in each shell, not what the shell itself might hold. We calculate the volume of each shell and add them all up to get the total volume.

The formula for the volume of a shell is $V = 2\ p\ r\ h\ w$
w is our dx or dy. The width and the height of the shell tell us which to use.

Q.18: How do we find the length of a curve?

Answer: Simply we find the length of a given curve by dividing the curve into very small segments and calculating the length of each segment. Then we add up the lengths found.

Mathematically we can say that measure the intervals by keeping the *Dx* same for each interval. Each segment approximates a straight line, so use the distance formula to find the length of each segment

Lecture # 39 Improper Integral

Q.19: What type of an integral is called when the integrand is not bounded and do not exist over a finite interval?

Answer: An integral exists if the integrand is both "bounded" and exists over a "finite" interval. If either of these conditions is not true then the integral is said to be "improper".

Q.20: Why do we replace the infinity with a variable solving the improper integrals?

Answer: Such type of integrals in which one or both of the limits of integration are infinity, the interval of integration is said to be over an infinite interval. We will replace the infinity with a variable (usually ***t***), do the integral and then take the limit of the result as ***t*** goes to infinity.

Q.21: When the limit of improper integral converges or diverges?

Answer: The value of the limits is the value assigned to the integral. If this limit exists, the improper integral is said to be **converge** i.e.we will call these integrals convergent if the associated limit exists and is a finite number (*i.e.* it is not plus or minus infinity).

If the limit does not exist, then the improper integral is called to **diverge**, in which case it is not assigned a value i.e. divergent if the associated limits either doesn't exist or is (plus or minus) infinity.

Lecture # 40 L'Hopital's Rule and Indeterminate Forms

Q.22: What is an indeterminate form?

Answer: An **indeterminate form** is an algebraic expression obtained in the context of limits. Limits involving algebraic operations are often performed by replacing sub expressions by their limits; if the expression obtained after this substitution does not give enough information to determine the original limit, it is known as an indeterminate form. The term indeterminate form is used to convey the idea that the limit cannot be determined without some additional work.

The indeterminate forms include 0^0, 0/0, 1^∞, ∞ - ∞, ∞/∞, 0×∞, and ∞^0.

Q.23: What happens if both the numerator and the denominator tend to 0?

Answer: Generally it cannot be calculated. Of course division by zero is never possible. The limit may or may not exist. The indeterminate form 0/0 is particularly common in calculus because it often arises in the evaluation of derivatives using their limit definition.

Mathematically this is an example of an indeterminate form 0/0. As *x* approaches 0, the ratios x/x^3, x/x, and x^2/x go to , 1, and 0 respectively. In each case, however, if the limits of the numerator and denominator are evaluated and plugged into the division operation, the resulting expression is 0/0. So 0/0 can be 0 or it can be and, in fact, it is possible to construct similar examples converging to any particular value. That is why the expression 0/0 is indeterminate. Since anything times 0 is 0, 0/0 is the set of numbers we are working with. So in this case **L'Hopital's rule** uses derivatives to help compute limits with indeterminate forms. Application of the rule often converts an indeterminate form to a determinate form, allowing easy computation of the limit.

Q.24: Why do we use L'Hopital's rule?

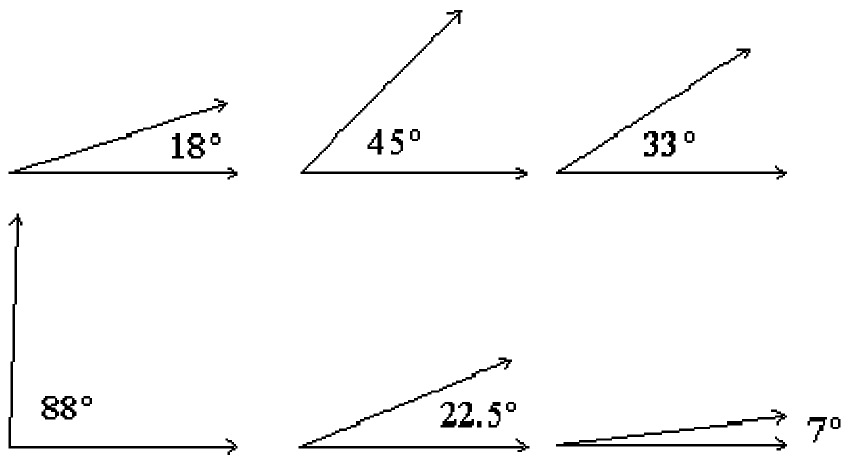
Answer:L'Hopital's rule allows us to replace a limit problem with another that may be simpler to solve. L'Hopital's rule uses derivatives to help compute limits with indeterminate forms.

Q.25: Acute Angles?

An acute angle is an angle measuring between 0 and 90 degrees.

Example:

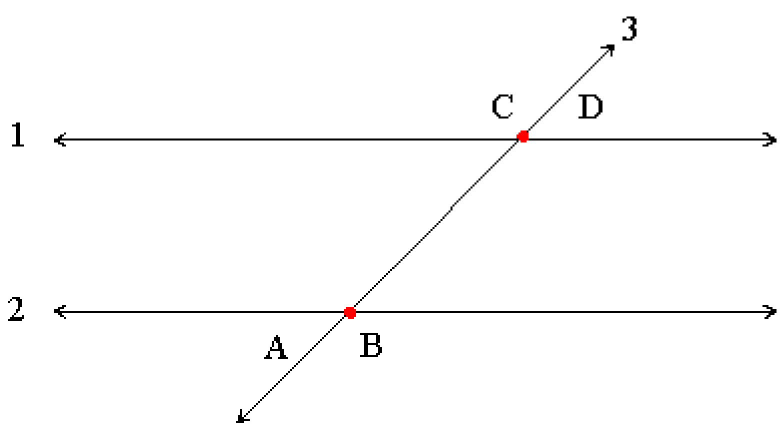
The following angles are all acute angles.



Q.26:Alternate Exterior Angles?

Alternate Exterior Angles

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle D are called alternate exterior angles. Alternate exterior angles have the same degree measurement. Angle B and angle C are also alternate exterior angles.



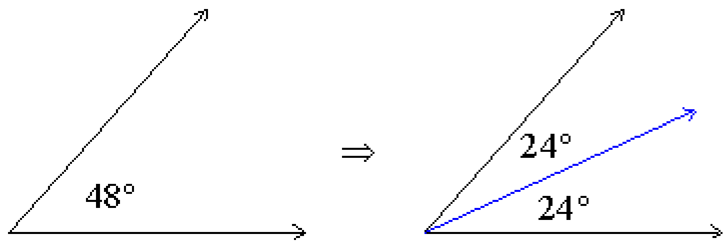
Q.27: Angle Bisector?

Angle Bisector:

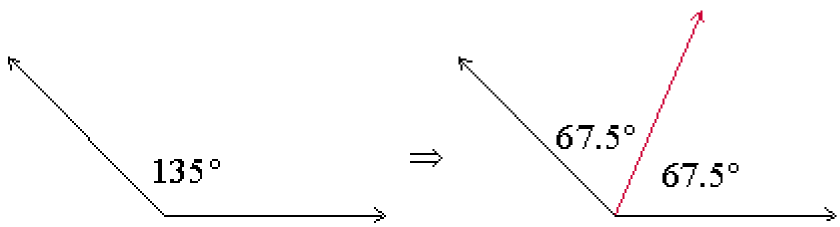
An angle bisector is a ray that divides an angle into two equal angles.

Example:

The blue ray on the right is the angle bisector of the angle on the left.



The red ray on the right is the angle bisector of the angle on the left.



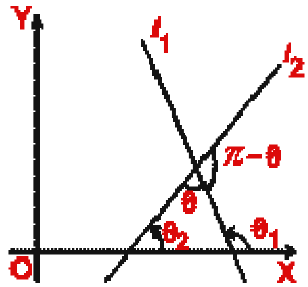
Q.28:The angle of inclination?

The angle of inclination:

The positive angle, less than 180°, measured from the positive .x-axis to the given line.

Let l_1 and l_2 be the two non-vertical and non-perpendicular lines with slopes m_1 and m_2 respectively. Let θ_1 and θ_2 be their inclinations, then m_1 = tan θ_1 and m_2 = tan θ_2. There are two angles θ and π- θ between the lines l_1 and l_2, given by

tan $\theta = \pm (m_1 - m_2)(1+m_1 m_2)$



This angle $\theta = \tan^{-1}(\frac{(m_1 - m_2)}{1 + m_1 m_2)})$ is always between 0° and 180°, and is

measured counterclockwise from the part of the *x*-axis to the right of the

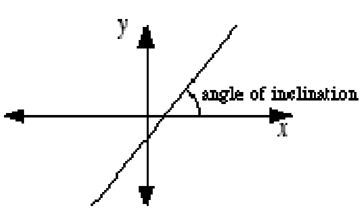
line.

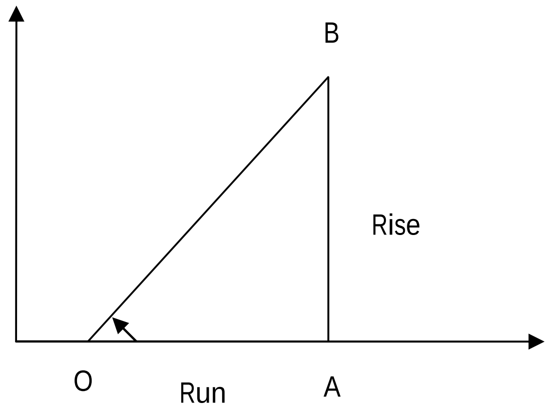
Q.29: Angle of Inclination of a Line?

Angle of Inclination of a Line

The angle between a line and the *x*-axis. This angle is always between 0° and 180°, and is measured counter clock wise from the part of the *x*-axis to the right of the line.

Note: All horizontal lines have angle of inclination 0°. All vertical lines have angle of inclination 90°. Also, the slope of a line is given by the tangent of the angle of inclination.





In the above figure, if we take slope of the line , then it will become

$$m= \frac{AB}{OA} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

In the above figure, you can see that if we take tan θ , then it will become

$$\text{Tan } \Phi = \frac{AB}{OA} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

From (i) and (ii), we can equate the equations, so we have

$$m= \tan \Phi$$

Q.30:ANTISYMMETRIC RELATION?

ANTISYMMETRIC RELATION:

Let R be a binary relation on a set A.R is anti-symmetriciff$\forall$a, b $\in$A if (a, b)

$\in$R and (b, a) $\in$R then a = b

Here you can see that

(a , b) $\in$R if we change the position of a and b i.e (b, a) then it should again belongs to R i.e (b, a) $\in$R

So by the definition of ant-symmetric

$\forall$a, b $\in$A if (a, b) $\in$R and (b, a) $\in$R then a = b.

<u>REMARK:</u>

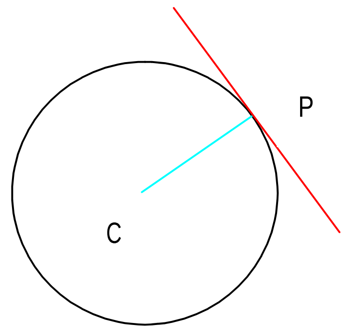
R is not anti-symmetric if there are elements a and b in A such that

(a, b) $\in$R and (b, a) $\in$R but a $\neq$ b

Q.31: Circle Problem?

Let P(2,3) be a point on circle with center (4,-1). Find the slope of the line that is tangent to the circle at P.

Here we need a diagram. But let me tell you about tangent to the circle; A tangent is a line that touches the circle only on a single point. Consider the diagram:



Let C be the center of the circle and P be the given point on the circle. Then we can find the slope of the line CP and then also that of the perpendicular line that is the tangent to the circle at P (shown with red color, tangent is always perpendicular to such line).

You solved as:

Let P(2,3) the point on circle with center C(4,-1) then the slope of line CP is given as

$$m = \frac{-1-3}{4-2} = -2$$

This is the slope of the line CP , now we are to find the slope of the tangent line. As this is perpendicular to the line CP so its slope is

-1/m= ½

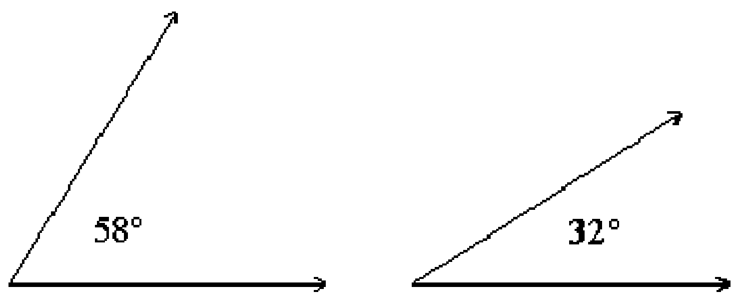
Q.32: Define Complementary Angles?

Complementary Angles

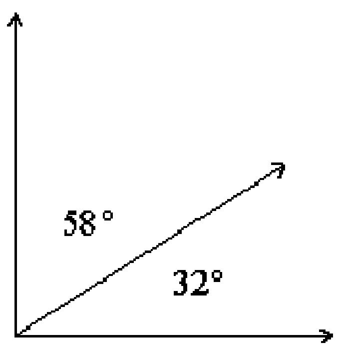
Two angles are called complementary angles if the sum of their degree measurements equals 90 degrees. One of the complementary angles is said to be the complement of the other.

Example:

These two angles are complementary.



Note that these two angles can be "pasted" together to form a right angle!



Q.33: Complex numbers?

Complex Numbers:

The number of the form z = a + ib where a and b are real numbers and “i” read as iota.

“a” is called the real part of the complex number and “b” is called the imaginary part of the complex number.

Every real number can be written in the form of complex number with 0 imaginary part.

For example 5 is a real number it become a complex number in this way 5 + 0i. Here 5 is the real part and 0 is the imaginary part.

Real numbers is a subset of complex numbers.

The set of complex number is denoted by C.

The complex number z = a + ib can be written as (a, b).

Q.34: Define Composition of function

Composition of function:-

In simple words:

When we operate an operator (operator may be addition or subtraction or any other) on any two functions we get a new function, that's mean we compose a new function.

For example:-

If we have two functions

F(x)=1+$\sqrt{x-2}$ and g(x)=x-1

If we use addition operator between two functions then, we have

F(x)+ g(x) =(1+$\sqrt{x-2}$)+(x-1)

=1+$\sqrt{x-2}$ + x-1

=x+$\sqrt{x-2}$

(That's mean we have compose a new function by adding two functions)

Another Example:-

If we have two functions

F(x)= x^3 and g(x)=x+4

(F∘g)(x)=f(g(x)) [By definition]

  =f(x+4)   [since   g(x) =x+4]

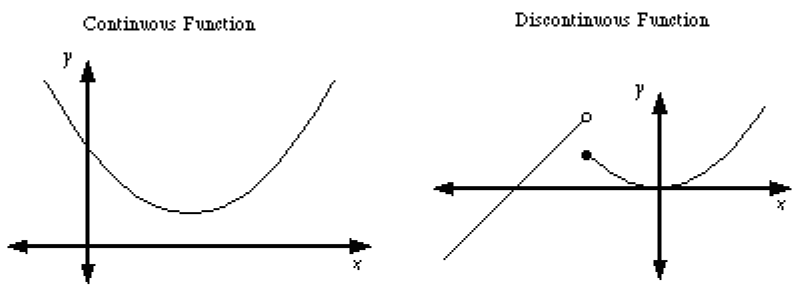
  = $(x+4)^3$ [We have replace x by x+4 in F(x) since F(x) = x^3]

That's mean we have compose a new function.

Q.35: Continuous Function?

Continuous Function

A function with a connected graph.

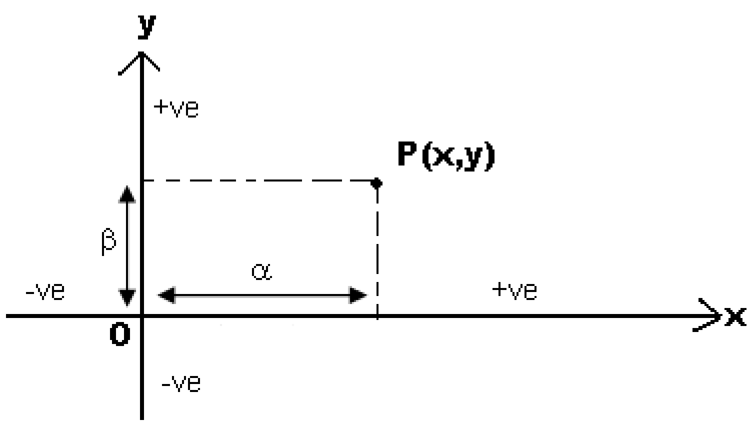


Definition of Continuity: A function is continuous at $x = a$ if

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Q.36: What's meant by Coordinates Lines?

In coordinate geometry, ox and oy are **two lines** intersecting at right angles at O, where Ox and Oy are the **coordinate axes or Coordinates Lines** and O is the origin. Any point lying to the right of Oy has a positive x-coordinate and any point above the Ox axis has a positive y-coordinate as indicated in fig below. Any point P can be determined if its distance from the x and y axis are given. The distance of a point P from the y-axis is called the x-coordinate of the point and its distance from the x-axis, the y-coordinate.



A point P whose x-coordinate is α and y-coordinate β is represented by P(α,β).

Q.37 Show that (x+5)-2x+x-5 =3x+5-5+2x-5x?

Answer: Let

L.H.S =(x+5)-2x+x-5

= x+5-2x+x-5

= 2x+5-2x-5

=0

R.H.S=3x+5-5+2x-5x

= 5x+5-5

= 0

So

(x+5)-2x+x-5 =3x+5-5+2x-5x

Since

L.H.S = R.H.S

Its does not mean that L.H.S =0 and R.H.S=0 then 0 will be equal to 0

It shows that since the value of L.H.S and R.H.S are equal so L.H.S = R.H.S

It may be possible that

L.H.S =5 and R.H.S=5

It shows that since the value of L.H.S and R.H.S are equal so L.H.S = R.H.S

Lecture # 31 Evaluating Definite Integral by Substitution:Approximation by Riemann Sums

Q.38# If the exact solution of a definite integral does not exist then how it will be evaluated?

Answer: In case where definite integral cannot be evaluated exactly, one must settle for a numerical approximation.

Q.39# Why **we cannot use the same upper and lower limits while using substitution in definite integral?**

Answer: When we substitute, we are changing the variable, so we cannot use the same upper and lower limits. We can either:

- Do the problem as an *indefinite integral* first, then use upper and lower limits later
- Do the problem throughout using the new variable and the new upper and lower limits

- Show the correct variable for the upper and lower limit during the substitution phase.

Differentiation: (Lectures 16 - 20)

Q.40: Is it true that a derivative of a given function can be evaluated by more than one way?

ANSWER: This is possible that a derivative of given function can be evaluated by more than one way but not always true.

For example, derivative of a following function can be evaluated by MANY methods three of them are shown below:

$y = (4x^2 – 1) (7x^3 + x)$

Method I: (Using Product Rule)

$dy/dx = d/dx [(4x^2 – 1) (7x^3 + x)]$

$= (4x^2 – 1) d/dx [7x^3 + x] + (7x^3 + x) d/dx (4x^2 – 1)$

$= (4x^2 – 1) (21x^2 +1) + (7x^3 + x) (8x)$

$= 140x^4 – 9x^2 -1$

Method II: (First take x common from second factor)

$y = (4x^2 – 1) (7x^3 + x)$

$= (4x^2 – 1) x (7x^2 + 1)$

$= x (4x^2 – 1) (7x^2 + 1)$

$dy/dx = d/dx [x (4x^2 – 1) (7x^2 + 1)]$

$= x (4x^2 – 1) d/dx [7x^2 + 1] + x (7x^2 + 1) d/dx$

$(4x^2 – 1) + (4x^2 – 1)(7x^2 + 1)$ d/dx [x]

$= x(4x^2 – 1)(14x) + x(7x^2 + 1)(8x) + (4x^2 – 1)(7x^2 + 1)(1)$

$= 56x^4 – 14x^2 + 56x^4 + 8x^2 + 28x^4 + 4x^2 - 7x^2 - 1$

$= 140x^4 – 9x^2 - 1$

Method III: (Multiplication first)

$y = (4x^2 – 1)(7x^3 + x)$

$= 28x^5 – 3x^3 - x$

$dy/dx = d/dx [(4x^2 – 1)(7x^3 + x)] = d/dx [28x^5 – 3x^3 - x]$

$= 140x^4 – 9x^2 - 1$

Observe that even by using different procedures, the solution is same.

Q.41: In the lectures, what does the letter ‘c’ stands for? a variable or a constant.

ANSWER: In the lectures, the letter ‘c’ stands for a constant.

Q.42: What is the derivative of constant function?

ANSWER: Derivative of a constant function is zero.

Q.43: How can we use differentiation to find the slope of the line?

ANSWER: As we know linear equation has a straight line as its graph. So, differentiation of linear equation of a given line evaluates slope of that line.

Slope of the line = m = dy / dx

Q.44: What is the difference between Differentiation and the Derivative?

ANSWER: Differentiation is the process of finding a derivative whereas Derivative is the result of performing the process of differentiation.

Q.45: Discuss differentiability of a function on an open interval.

ANSWER: The function is said to be **differentiable** at a given point if the derivative exists at that point. Also, the function is differentiable on an open interval if the derivative exists at every point on that interval.

Q.46: In how many steps can we find the derivative of function f by 1st principle (Abinitio Method, by definition)?

ANSWER:

There are three basic steps:

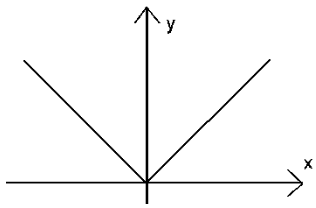
1. Write an expression for y = f(x) and y + Δy = f(x + Δx).
2. Subtract equations in step 1 and simplify Δy = f(x + Δx) – f(x).
3. Divide both sides of equation in step 2 by Δx and get the derivative by evaluating the limit as Δx approaches 0.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Q.46: What is the relationship between the continuity and differentiability of a function f at any point 'a' of its domain?

ANSWER: If *f* is differentiable at any point 'a' then *f* is also continuous at 'a'.

But a function continuous at a point 'a' may not be differentiable there. For example, whenever the graph of a function has a corner at a point, no break or gap there, the function is continuous at that point but not differentiable. For example, $y = |x|$ is continuous at $x = 0$, but it is not differentiable there.

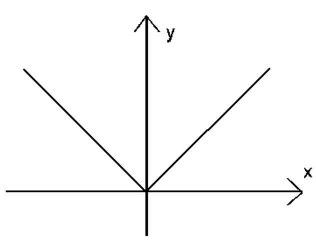
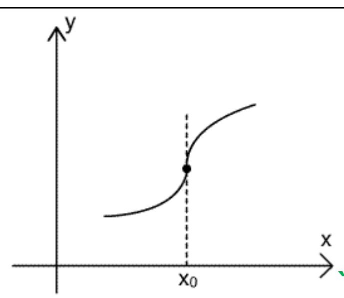
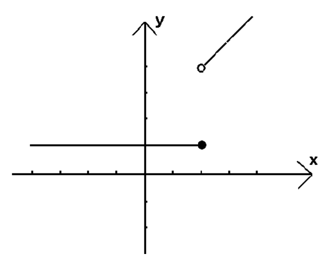


graph of f(x) = |x|

Q 47. Discuss the non-existence of derivative of a function at a point?

<u>ANSWER:</u> The most commonly encountered points of non-differentiability can be classified as:

1. Corners
2. Vertical Tangents
3. Points of discontinuity

 *Graph of* $f(x) = \|x\|$ Corner at x = 0 so not differentiable although the graph is continuous there.	 At x_0 the tangent line is vertical so slope is undefined and function is not differentiable.	 Graph of g(x) = { 1 x ≤ 2 x + 2 x > 2 Discontinuous at x = 2 and so not

		differentiable there.
Example of Corner	**Example of Vertical Tangents**	**Example of Point of discontinuity**

Q 48. Can we obtain 2nd derivative of a function by squaring first derivative of that function?

ANSWER: No $\left(\frac{dy}{dx}\right)^2$ and $\frac{d^2y}{dx^2}$ have different meanings.

$\left(\frac{dy}{dx}\right)^2$ is obtained by squaring first derivative of the function whereas $\frac{d^2y}{dx^2}$ is evaluated by again differentiating first derivative of a function. Occasionally one may encounter a situation where 2nd derivative of a function $\frac{d^2y}{dx^2}$ and square of first derivative of a function $\left(\frac{dy}{dx}\right)^2$ are same but that does not mean that both of them are always equal for all functions.

Q 49. Is it true that derivative of *f(x)* = *b*^{x}, where *b* is a constant and *x* is a variable, is *xb*$^{x-1}$

$$Let\ \ y = b^x\ \ so\ that$$

$$\ln y = \ln b^x = x \ln b$$

ANSWER: No. $\frac{1}{y}\frac{dy}{dx} = \ln b \qquad Differentiating\ both\ sides$

$$\frac{dy}{dx} = y \ln b$$

$$\frac{dy}{dx} = b^x \ln b \qquad Putting\ value\ of\ \ y$$

Q 50: Discuss the derivative of base *b* logarithmic function (*log$_b$x*)

and natural logarithm function (*lnx*)?

For *b*> 0 and *x*> 0, derivative of base *b* logarithmic function $(\log_b x)$ is:

$$\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}$$

Natural logarithm is special case of base *b* logarithmic function where *b* = *e*, thus

$$\frac{d}{dx}[\log_e x] = \frac{d}{dx}[\ln x] = \frac{1}{x \ln e}$$

$As\ \ln e = 1, so$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Q. 51: How angle in radians is converted to degrees and vice versa?

ANSWER: Converting Radians to Degrees:

To convert radians to degrees, multiply by 180/ Π:

degrees = radians * (180 / Π)

Converting degrees to radians:

To convert degrees to radians, multiply by Π/180:

radians = degrees * (Π / 180)

Q. 52: Is there any easy way of remembering the derivative of trigonometric functions?

ANSWER:

Let us agree to call the trigonometric functions as follows:

FUNCTION	CO-FUNCTION

	(Function names starting with co)
sin x	*cos x*
tan x	*cotan x*
sec x	*cosec x*

Memorize the derivatives of FUNCTION. Derivatives of any CO-FUNCTION can be obtained from the derivative of the corresponding FUNCTION by introducing a minus sign and replacing each FUNCTION in the derivative by its CO-FUNCTION.

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cot x) = -\cos ec^2\ x$
$\frac{d}{dx}(\sec x) = (\sec x)(\tan x)$	$\frac{d}{dx}(\cos ec\ x) = -(\cos ec\ x)(\cot x)$

Maximum and Minimum Values of a Function (Lecture 22 – 23):

Q.53: What is the difference between local maximum and relative maximum value of a function?

ANSWER: Local maximum and relative maximum are synonyms, that is, different words for same concept. Similarly local minimum is also named as relative minimum.

Q.54: What is the difference between critical point and stationary point of the function?
ANSWER: The term "critical point" is often confused with "stationary point".

Stationary point x_0 is the point, at which the derivative of a function $f(x)$ vanishes,

$f'(x_0) = 0$
That is, a point where the function "stops" increasing or decreasing (hence the name).
Critical point x_0 of a function $y = f(x)$ is the point at which $f'(x_0) = 0$
Or

$f(x)$ is not differentiable at x_0

Critical point is more general, it is *either* a stationary point *or* a point where the derivative is not defined.A stationary point is always a critical point, but a critical point is not always a stationary point as it may also be a non-differentiable point.

Q .55: If *c* is a critical point of a function *f* then is it always true that *f*(*c*) is either a local maximum or a local minimum of *f*?
ANSWER: No, a critical point may or may not be a local maximum or local minimum of the function. For example, $f(x) = x^3$ has a critical point at $x = 0$ yet $f(0)$ is neither a local maximum nor a local minimum.

Q.56: Distance formula problem?

A point (x, y) moves so that its distance to (2, 0) is $\sqrt{2}$ times its distance to (0,1)

(a). Show that the point moves along a circle

(b). find the center and radius

Solution:

(a) By using distance formula

Distance of point to (2, 0) from (x, y) = $\sqrt{(x-2)^2+(y-0)^2}$

Distance of point to (0,1) from (x, y)= $\sqrt{(x-0)^2+(y-1)^2}$

By given condition

Distance to (2, 0) is $\sqrt{2}$ times its distance to (0,1)

So

$\sqrt{(x-2)^2+(y-0)^2} = \sqrt{2}\,\sqrt{(x-0)^2+(y-1)^2}$

Squaring both sides we get

$$\left((x-2)^2+(y-0)^2\right)=2\left((x-0)^2+(y-1)^2\right)$$

on simplification

$$x^2+4x+y^2-4y=2$$

As we done in above question, same procedure

By completing square

$$(x+2)^2+(y-2)^2=10$$

$$\text{or} \quad (x+2)^2+(y-2)^2=\left(\sqrt{10}\right)^2$$

$$(x-(-2))^2+(y-2)^2=\left(\sqrt{10}\right)^2$$

Since this is the equation of the circle, and by the definition of circle, (The distance from a fixed point to a locus (movable point) is always constant)

It mean that the point (-2, 2) will always point on along the circle having the fixed radius $\sqrt{10}$

And $\Rightarrow$ *center* $(-2,2)$ *and radius* $\sqrt{10}$

Concavity: (Lecture # 21)

Q.57: **How differentiability property is used to find whether the function f is concave up or concave down in the given interval?**

ANSWER:a. *f* is **concave up** on the given interval if *f'* is increasing on the interval **OR** *f* "(*x*) > 0.

b. *f* is **concave down** on the given interval if *f* ' is decreasing on the interval **OR** *f* "(*x*) < 0.

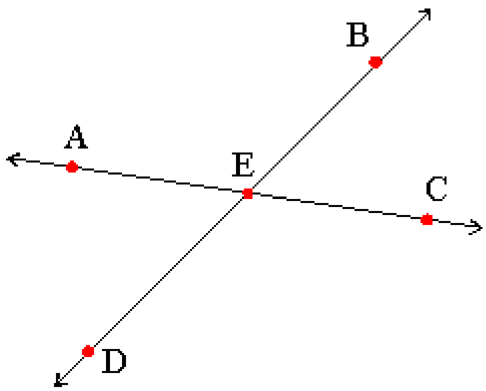
Q. 58: If a function f is concave down on interval (-∞, 0) and concave up on (0, ∞) then what about the point x = 0?

ANSWER: The point *x* = 0 is called an **inflection point**, that is, a point where the function is changing from being concave up to concave down or vice versa. Inflection points may be stationary points but are not relative maxima or relative minima.

Q.59: Define Vertical Angles?

Vertical Angles

For any two lines that meet, such as in the diagram below, angle AEB and angle DEC are called vertical angles. Vertical angles have the same degree measurement. Angle BEC and angle AED are also vertical angles.



Q.60: Explain The Slope formula with example ?

The Slope formula (Slope of the line between two points) between any two points A (x_1, y_1) and B (x_2, y_2) in the space is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Where “m” is the slope of the line.

Example: Find the slope of the line between the two points A (1, 1) and B (-2,-8)

Solution:

Slope formula between the two points is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Putting the values (as by the giving point s)

Thus $m = \frac{-8-1}{-2-1} = 3$

$m = \frac{-9}{-3} = 3$

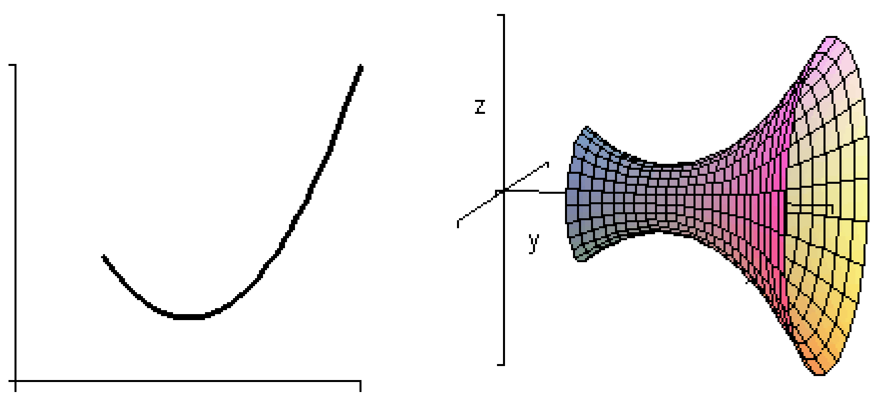
$m = 3$

Thus slope of the line between the two points is $m = 3$

Q.61:Explain Surface Area?

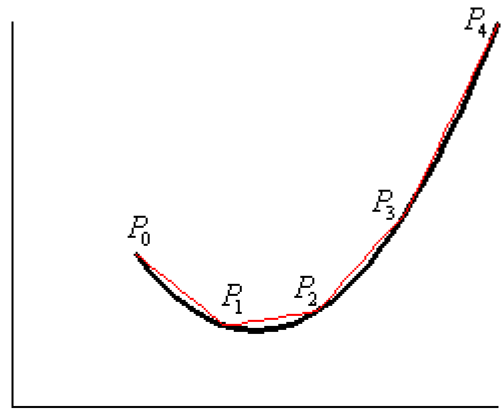
Surface Area

In this section we want to find the surface area of this region. Let us consider, rotating the continuous function $y = f(x)$ in the interval $[a,b]$ about the *x*-axis. Below is a sketch of a function and the solid of revolution we get by rotating the function about the *x*-axis.

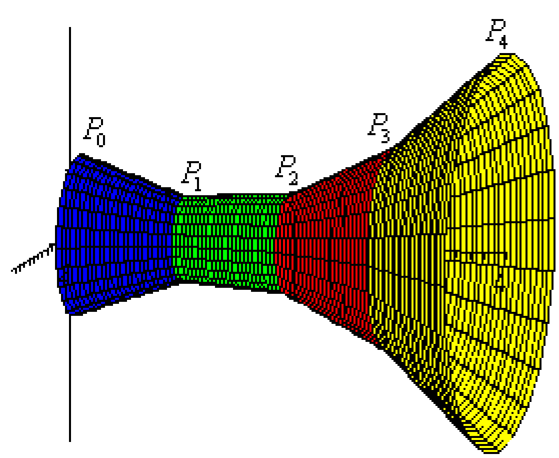


We can derive a formula for the surface area much as we derived the formula for arc length We will start by dividing the integral into *n* equal subintervals of width Δx. On each subinterval we will approximate the function with a straight line that agrees with the function at the endpoints of the each interval. Here is a sketch of that for our

Representative function using *n=4*.



Now, rotate the approximations about the *x*-axis and we get the following solid.



The approximation on each interval gives a distinct portion of the solid and to make this clear each portion is colored differently. Each of these portions is called frustums and we know how to find the surface area of frustums.

The surface area of a frustum is given by,

$$A = 2\pi r l$$

Where,

$$r = \frac{1}{2}(r_1 + r_2) \qquad r_1 = \text{radius of right end}$$

$$r_2 = \text{radius of left end}$$

l is the length of the slant of the frustum

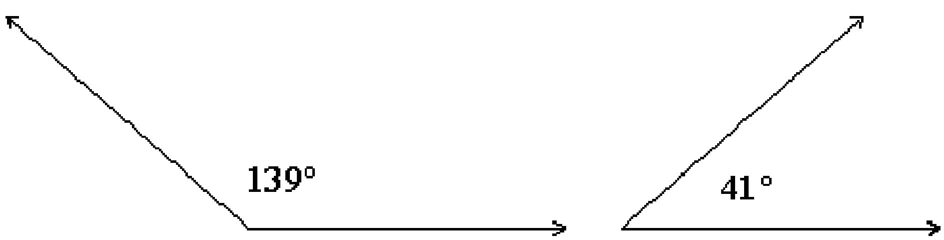
Q. 62:Explain Supplementary Angles?

Supplementary Angles

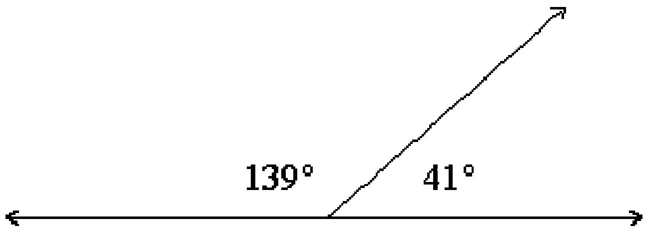
Two angles are called supplementary angles if the sum of their degree measurements equals 180 degrees. One of the supplementary angles is said to be the supplement of the other.

Example:

These two angles are supplementary.



Note that these two angles can be "pasted" together to form a straight line!



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