Mth501 subjective for mid term
Latest spring 2013
By
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$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q1. Applying certain elementary row operation to the elementary matrix to produce an identity matrix $I_{3\times 3}$. [2 marks]

Solution:

Multiply R_2 by 1/5

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We get the identity matrix $I_{3\times 3}$

Q2. Why is it NOT possible to solve the following system of linear equations applying the Cramer’rule? [2 marks]

$$\begin{aligned} 3x_1 + 2x_2 &= 10 \\ 9x_1 + 6x_2 &= 30 \end{aligned}$$

Solution:

$$let\ A=\begin{bmatrix}3 & 2\\ 9 & 6\end{bmatrix}, x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}, b=\begin{bmatrix}10\\ 30\end{bmatrix}$$

$$\det(A)=\begin{bmatrix}3 & 2\\ 9 & 6\end{bmatrix}=18-18=0$$

$$\det(A)=0$$

since it is an singular matric and its determinant is zero.

we need determinant of A f.or applying crammers rule

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Q3. Determine whether or not the inverse of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 1 & 0 & 0\end{bmatrix}$ **exists? Justify your answer with appropriate reason. [3 marks]**

Solution:

$$|A|=\begin{vmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 1 & 0 & 0\end{vmatrix}$$

$$|A|=1(0-0)+2(0-6)-3(0-4)$$

$$|A|=1(0)+2(-6)-3(-4)$$

$$|A|=0+(-12)+12$$

$$|A|=0$$

As the determinant of the given matrix is zero its mean it is a singular matrix the inverse of the singular matrix does not exist.

Q4. Compute $\det(A)$ **by using a cofactor expansion across the third row, where** $A=\begin{bmatrix}3 & 2 & 1\\ 2 & 1 & 2\\ 0 & 2 & 0\end{bmatrix}$ **.**

[3 marks]

Solution:

Using cofactor expansion along the first column:

$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 0 & 2 & 0 \end{vmatrix} = (0)(-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (2)(-1)^{2+1}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + (0)(-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

now if we compare it with the formula

det A= $(0)C_{31} + (2)C_{32} + (0)C_{33}$

$$=(0)(-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (2)(-1)^{2+1}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + (0)(-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$=0+(2)(-1)^{3}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 0$$

=0+ {-2(6-2)}+0

=0-2(4)+0

= -8

$$A = \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 0 \\ \hline 0 & 0 & 2 \end{array}\right] \qquad B = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Q5. Compute AB using block multiplication, where a .

[5 marks]

Solution:

Let

$$A=\begin{bmatrix}1&2&1\\3&4&0\\0&0&2\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}, B=\begin{bmatrix}1&2&3&1\\4&5&6&1\\0&0&0&1\end{bmatrix}=\begin{bmatrix}B_{11}&B_{12}\\B_{21}&B_{22}\end{bmatrix}$$

let

$$A_{11}=\begin{bmatrix}1&2\\3&4\end{bmatrix}, A_{12=}\begin{bmatrix}1\\0\end{bmatrix}, A_{21}=\begin{bmatrix}0&0\end{bmatrix}, A_{22}=\begin{bmatrix}2\end{bmatrix}$$

$$B_{11}=\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}, B_{12}=\begin{bmatrix}1\\1\end{bmatrix}, B_{21}=\begin{bmatrix}0&0&0\end{bmatrix}, B_{22}=\begin{bmatrix}1\end{bmatrix}$$

now

$$AB=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}\begin{bmatrix}B_{11}&B_{12}\\B_{21}&B_{22}\end{bmatrix}=\begin{bmatrix}A_{11}B_{11}+A_{12}B_{21}&A_{11}B_{12}+A_{12}B_{22}\\A_{21}B_{11}+A_{22}B_{21}&A_{21}B_{12}+A_{22}B_{22}\end{bmatrix}$$

the number of colums of A equals numbers of rows of B

so we can performed multiplication operation:

$$A_{11}B_{11}+A_{12}B_{21}=\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0&0&0\end{bmatrix}=\begin{bmatrix}9&12&15\\19&26&33\end{bmatrix}$$

$$A_{11}B_{12}+A_{12}B_{22}=\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\end{bmatrix}=\begin{bmatrix}4\\7\end{bmatrix}$$

$$A_{21}B_{11}+A_{22}B_{21}=\begin{bmatrix}0&0\end{bmatrix}\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}+\begin{bmatrix}2\end{bmatrix}\begin{bmatrix}0&0&0\end{bmatrix}=\begin{bmatrix}0&0&0\end{bmatrix}$$

$$A_{21}B_{12}+A_{22}B_{22}=\begin{bmatrix}0&0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}2\end{bmatrix}\begin{bmatrix}1\end{bmatrix}=\begin{bmatrix}2\end{bmatrix}$$

$$AB=\begin{bmatrix}A_{11}B_{11}+A_{12}B_{21}&A_{11}B_{12}+A_{12}B_{22}\\A_{21}B_{11}+A_{22}B_{21}&A_{21}B_{12}+A_{22}B_{22}\end{bmatrix}=\begin{bmatrix}9&12&15&4\\19&26&33&7\\0&0&0&2\end{bmatrix}$$

Q6. Consider the vector space $V=R^3$ **and the set** W **consists of all vectors in** R^3 **whose entries**

are equal : that is ,

$$W = \{(a, b, c) : a = b = c\}$$

Show that W is a subspace of R^3. [5 marks]

Solution:

To check if W is the subspace of R^3 ,we 1st check that axiom 1 and 6 of a vector space holds.

Let

$u = (a_1, b_1, c_1)$ and v=(a_2, b_2, c_2)be vectors in W then

u+v=$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$is in W

and also if k ia any scalar and u=(a_1, b_1, c_1)in any vector W,then ku=k$(a_1, b_1, c_1)^{s}$

Hence W is a subspace.

MIDTERM EXAMINATION
Spring 2013
(MTH501- Linear Algebra (Session - 3)

Q1: If A = B, then determine the values of x and y; where

$$A = \begin{bmatrix} 1 & y+2 \\ x+2 & 3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

Solution:

As we know that matrix A=B

its mean that the every entry in A = to the coresponding entry in B

so its clearly seen that value of

x+2=4 .....x=4/-2=x=-2

y+2=2......y=-1

Q2: Determine which of the following condition(s) hold(s) for a vector space V over R. Justify your answer with appropriate reason.

$$a)\left\{\overrightarrow{x}+\overrightarrow{y}\mid\overrightarrow{x}\in V,\overrightarrow{y}\in V\right\}=V$$

$$b)\left\{\overrightarrow{x}+\overrightarrow{y}\mid\overrightarrow{x}\in V,\overrightarrow{y}\in V\right\}=VxV$$

$$c)\left\{\lambda\overrightarrow{x}\mid\overrightarrow{x}\in V,\lambda\in R\right\}=RxV$$

Solution:

(b) and (c) both are truefor vector space V over a field R is a set V equipped with an operation called (vector) addition, which takes vectors u and v and produces another vector .

There is also an operation called scalar multiplication, which takes an element and a vector and produces a vector .

Q3: Determine whether or not the solution of the following system of linear equations is possible using inversion algorithm? Justify your answer with appropriate reason.

$$2x_1+4x_2+3x_3=3$$
$$4x_1+8x_2+6x_3=4$$
$$6x_1+12x_2+9x_3=4$$

Solution:

let

$$A=\begin{bmatrix}2 & 4 & 3\\ 4 & 8 & 6\\ 6 & 12 & 9\end{bmatrix}, x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}, b=\begin{bmatrix}3\\ 4\\ 4\end{bmatrix}$$

$$\det(A)=2(72-72)-4(36-36)+3(48-48)$$

$$\det(A)=2(0)-4(0)+3(0)$$

$$\det(A)=0$$

since matrix A is non invertible matrix so we can not apply inversion algorithm here.

Q4: Find a Matrix A such that W=ColA

$$W=\left\{\begin{bmatrix}2b+2c\\ a+b-2c\\ 4a+b\\ 3a-b-c\end{bmatrix}; a,b,c \text{ are real}\right\}$$

Solution:

1st we write as a set of linear combinations:

$$w=\{a\begin{bmatrix}0\\1\\4\\3\end{bmatrix}+b\begin{bmatrix}2\\1\\1\\-1\end{bmatrix}+c\begin{bmatrix}2\\-2\\0\\-1\end{bmatrix}: a,b,c \text{ in R}\}=\text{span }\left\{\begin{bmatrix}0\\1\\4\\3\end{bmatrix},\begin{bmatrix}2\\1\\1\\-1\end{bmatrix},\begin{bmatrix}2\\-2\\0\\-1\end{bmatrix}\right\}$$

let

$$A=\begin{bmatrix}0 & 2 & 2\\1 & 1 & -2\\4 & 1 & 0\\3 & -1 & -1\end{bmatrix}$$

Q5: Find an LU – decomposition of the Matrix

$$A=\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix} \qquad \begin{bmatrix}* & 0\\* & *\end{bmatrix}$$

$$=\begin{bmatrix}1 & \frac{3}{2}\\5 & 7\end{bmatrix} multipler=\frac{1}{2} \qquad \begin{bmatrix}2 & 0\\* & *\end{bmatrix}$$

$$=\begin{bmatrix}1 & \frac{3}{2}\\0 & -\frac{1}{2}\end{bmatrix} multipler=-5 \qquad \begin{bmatrix}2 & 0\\5 & *\end{bmatrix}$$

$$=\begin{bmatrix}1 & \frac{3}{2}\\0 & 1\end{bmatrix} multipler=-2 \qquad \begin{bmatrix}2 & 0\\5 & 1\end{bmatrix}$$

Q6: Consider the vector space V=R^2 and the set W consists of all points in R^2 such that, W={(a,b) : a,b ≥ 0}Show that W is not a subspace of R^2.

Solution:

This is not subspace because it is not enclosed under scalar multiplication. So ,Vector space $V=R^2$ where R^2 not passing through origin is not a subspace of $R^{2.}$

MIDTERM EXAMINATION
Spring 2013
MTH501- Linear Algebra (Session - 2)

Q1. If $A=\left[\begin{array}{cc|cc} 1 & 3 & 2 & 4 \\ \hline 2 & 4 & 1 & 3 \\ 5 & 3 & 2 & 4 \\ \hline 9 & 8 & 6 & 5 \end{array}\right]$ and $B=\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 3 & 6 \\ 4 & 5 \end{bmatrix}$ then partition B in such a way that the multiplication can be possible?

Solution:

No as matrix A has 6 partitions it will only able to multiply with matrix B if ad only if:
No of columns of A =no of rows of B

Q2. File the determinant and tell that the given matrix is singular or no singular

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution:

Det (A)=1(0-0)-2(0-6)+3(0-4)

Det (A)=1+12-12

Det (A)=1 it is non singular matrix

Q3. Determine whether the inverse in possible or not of the give matrix and justify your answer

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 1 & 0 & 0\end{bmatrix}$$

Solution:

As

Det (A)=1(0-0)-2(0-6)+3(0-4)

Det (A)=1+12-12

Det (A)=1 it is non singular matrix

Determinant of the matrix is non singular so its inverse is possible.

Q4. Apply Cramer's rule and find the inverse of $A=\begin{bmatrix}2 & 3\\ -1 & 5\end{bmatrix}$

Solution:

For the matrrix say

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} \Rightarrow \det A = 10 - (-3) = 13$$

$\Rightarrow A^{-1}$ will also be a 2×2 matrix

As

$Aji =$ submatrix of A formed by deleting row ***j*** and column ***i***

So in this case

$A_{11} =$ submatrix of A formed by deleting row ***1*** and column ***1*** $= [5]$

$A_{12} =$ submatrix of A formed by deleting row ***1*** and column ***2*** $= [-1]$

$A_{21} =$ submatrix of A formed by deleting row ***2*** and column ***1*** $= [3]$

$A_{22} =$ submatrix of A formed by deleting row ***2*** and column ***2*** $= [2]$

and

$$\det A_i(e_j) = (-1)^{i+j} \det(A_{ji}) = C_{ji}$$

where e_j is the jth column of identity matrix $I_{n\times n}$

So in this case

$$C_{11} = \det A_1(e_1) = (-1)^{1+1} \det A_{11} = (+1)\det[5] = 5$$

$$C_{12} = \det A_2(e_1) = (-1)^{1+2} \det A_{12} = (-1)\det[-1] = (-1)(-1) = 1$$

$$C_{21} = \det A_1(e_2) = (-1)^{2+1} \det A_{21} = (-1)\det[3] = -3$$

$$C_{22} = \det A_2(e_2) = (-1)^{2+2} \det A_{22} = (+1)\det[2] = 2$$

By Cramer's rule,

$$\{(i,j) - entry\ of\ A^{-1}\} = x_{ij} = \frac{\det A_i(e_j)}{\det A} = \frac{C_{ji}}{\det A}$$

So for the current matrix;

$$\{(1,1) - entry\ of\ A^{-1}\} = x_{11} = \frac{\det A_1(e_1)}{\det A} = \frac{C_{11}}{\det A} = \frac{5}{13}$$

$$\{(1,2) - entry\ of\ A^{-1}\} = x_{12} = \frac{\det A_1(e_2)}{\det A} = \frac{C_{21}}{\det A} = \frac{-3}{13}$$

$$\{(2,1) - entry\ of\ A^{-1}\} = x_{21} = \frac{\det A_2(e_1)}{\det A} = \frac{C_{12}}{\det A} = \frac{1}{13}$$

$$\{(2,2) - entry\ of\ A^{-1}\} = x_{22} = \frac{\det A_2(e_2)}{\det A} = \frac{C_{22}}{\det A} = \frac{2}{13}$$

Hence by using equation # 4, we get

$$A^{-1} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} \frac{C_{11}}{\det A} & \frac{C_{21}}{\det A} \\ \frac{C_{12}}{\det A} & \frac{C_{22}}{\det A} \end{bmatrix} = \frac{1}{\det A}\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & \frac{-3}{13} \\ \frac{1}{13} & \frac{2}{13} \end{bmatrix}$$

Q5. If $A = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} \\ 3 & -2 \end{bmatrix}$ then show that B is multiplicative of A?

Solution:

$$AB=\begin{bmatrix}4 & 5\\ 6 & 6\end{bmatrix}\begin{bmatrix}-\frac{7}{2} & \frac{5}{2}\\ 3 & -2\end{bmatrix}$$

$$AB=\begin{bmatrix}\frac{47}{5} & 0\\ \frac{48}{5} & 3\end{bmatrix}\neq\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

As AB is not equal to I so ,A is not multiplicative identity of B

MIDTERM EXAMINATION
Spring 2010
(MTH501- Linear Algebra (Session - 3)

Q1. Find vector and parametric equation of the plane that passes through the origin of R^3 and is parallel to the vectors V_1= (1, 2, 5) and V_2 = (5, 0, 4).

Solution:

As vector equation of the plane passing through origin is $x = t_1 v_1 + t_2 v_2$

Let x = (x, y, z) then this equation can be expressed in component form as

(x, y, z) = t_1 (1, 2, 5) + t_2 (5,0,4)

This is the **vector equation of the plane**.

Equating corresponding components, we get

$x = t_1 + 5t_2$, $y = 2t_1$, $z = 5t_1 + 4t_2$

These are the **parametric equations of the plane.**

Q2. Which of the following is true? If V is a vector space over the field F.(justify your answer)

$a)\ \{\frac{x+y}{x}\ \varepsilon$ V,y ε V}=V

b) $\{\frac{x+y}{x}\ \varepsilon$ V,y ε V}=VxV

c) $\{\frac{\lambda V}{V}\varepsilon$ V,$\lambda\varepsilon$ F}FxV

Solution:

(b) and (c) both are correct vector space V over a field F is a set V equipped with an operation called (vector) addition, which takes vectors u and v and produces another vector .

There is also an operation called scalar multiplication, which takes an element and a vector and produces a vector .

$$let$$

Q3. $$v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} \text{and y=} \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$$

for what value$(s)$of h is y in the plane is generated by

$v_1$and v_2 ?

Solution:

we can write in matrix form as

$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ -2 & 7 & -5 \end{bmatrix}$$

$$2R_1 + R_3 \longrightarrow R_3' \begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 3 & 2h-5 \end{bmatrix}$$

$$2R_2 + R_1 \longrightarrow R_1' \begin{bmatrix} 1 & 0 & h-6 \\ 0 & 1 & -3 \\ 0 & 3 & 2h-5 \end{bmatrix}$$

$$-3R_2 + R_3 \longrightarrow R_3' \begin{bmatrix} 1 & 0 & h-6 \\ 0 & 1 & -3 \\ 0 & 0 & 2h+4 \end{bmatrix}$$

for h=2

y is in the plane generated.

Q8.given A and b ,write the augmented matrix for the linear system that corresponds to the matrix equation Ax=b. then solve the system and write the solution as a vector.

$$A=\begin{bmatrix}1 & 2 & 1\\ -3 & -1 & 2\\ 0 & 5 & 3\end{bmatrix}, b=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$$

Solution:

we can write the given ab in the matrix equation form Ax=b

$$Ax=\begin{bmatrix}1 & 2 & 1\\ -3 & -1 & 2\\ 0 & 5 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}=b$$

or

$$\begin{bmatrix}1 & 2 & 1\\ -3 & -1 & 2\\ 0 & 5 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$$

Q9. Q5.Find the AREA of parallelogram of the vertices (1,2,4)(2,4,-7) and (-1,-3,20). PG# 239